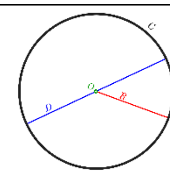
Circles, Midpoint and Distance

A circle is defined as a **two-dimensional figure**, which is round in shape where all the points on the surface of the circle are equidistant from the center point is called “C”. The distance from point C to the outside of the circle is called the radius of the circle (r). We have 3 formulas that we use to find the pieces to develop a formula for a circle.

Circle formula*:* *r (radius), center(h,k)*	**Distance formula**	**Midpoint formula:** (center at (h,k))
$r^2 = (x-h)^2 + (y-k)^2$	d=$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Example 1. Given the center and the radius

If you are given the center (5, -2) and the radius of 3, then you just need to substitute them into the circle formula. In the formula, the center is designated as (h,k) and the radius is r.

$$\boldsymbol{r^2 = (x-h)^2 + (y-k)^2}$$

$(3)^2 = (x-5)^2 + (y-(-2))^2$, then simplify

$$\boldsymbol{9 = (x-5)^2 + (y+2)^2}$$

Example 2. Given the center and one point on the circle.

If you are given the center and one point on the circle, you have a two-step process to get the equation of the circle. You always need the radius and the center. Since we are given the center we need to determine the radius. Since the radius is the distance from the center to any point on the circle, we can use the distance formula.

You are given that the center is at (3,4) and a point on the circle is (5,5). Thus, we can label the first point as (x_1, y_1) and the second as (x_2, y_2) to help us as we substitute the values into the distance formula.

d=$\sqrt{\boldsymbol{(x_2-x_1)^2 + (y_2-y_1)^2}}$

d = $\sqrt{(5-3)^2 + (5-4)^2}$, then simplify

d=$\sqrt{(2)^2 + (1)^2}$

$\sqrt{5}$ d=

this distance is our radius. Now we have both the center and radius so we can create our circle formula that fits the information we were given:

$(\sqrt{5})^2 = (x-3)^2 + (y-4)^2$, or simplified

5 = $\boldsymbol{(x-3)^2 + (y-4)^2}$

Example 3. Given two points that are the endpoints of the diameter

If you are given two points on the circle that lie on the endpoints of the diameter, you will need to calculate both the center **(h,k)** and the radius **(r).** To calculate the center, you can use the midpoint formula, since the radius is exactly in the middle between two points on the circle. Before you start substituting the points it is a good practice to label the coordinates as (x_1, y_1) and (x_2, y_2). If we start with the points (4,5) and (-2, -3) as points on the circle, we would have

$$\left(\frac{\boldsymbol{x_1+x_2}}{\boldsymbol{2}}, \frac{\boldsymbol{y_1+y_2}}{\boldsymbol{2}}\right)$$

$\left(\frac{4+(-2)}{2}, \frac{5+(-3)}{2}\right)$, then simplify

(1, 1) so this is our center **(h,k)**

Next, we need the radius (r). Since this is a distance, we can utilize the distance formula. However, the distance between two points on the circle gives us the diameter so we will need to use the fact that r = ½ d.

d=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d=$\sqrt{(-2-4)^2 + (-3-5)^2}$ **; then simplify**

d=$\sqrt{(-6)^2 + (-8)^2}$ d=$\sqrt{36 + 64}$ **d =**

$\sqrt{100}$ d = 10

thus, our radius = ½ (10) or 5.

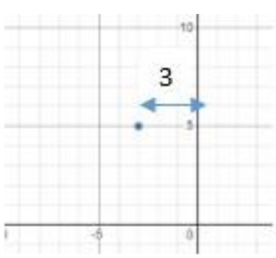
Now we have enough information to write the equation of the circle using the information we were given and what we calculated. R = 5 and the center (h,k) = (1,1)

$5^2 = (x-1)^2 + (y-1)^2$. simplified we have

$$\mathbf{25 = (x-1)^2 + (y-1)^2}$$

Example 4. Given the center and Tangent to an axis

You have a circle with a center of (-3,5) and it is **tangent** to the y- axis, and you need the equation of the circle. The best strategy is to graph the information given to determine the radius. The you can see that it is 3 steps from the y-axis. Since being tangent means, it only touches at 1 point, you can use this as your radius. Now we have all the information we need to create the equation of the circle.



$$r^2 = (x-h)^2 + (y-k)^2$$

$$(3)^2 = (x-(-3))^2 + (y-5)^2\text{, simplified}$$

$$\mathbf{9 = (x+3)^2 + (y-5)^2}$$

Find the equation of the circle for each of the following:

1. Center at (0, 3) and radius = 5
2. Center at (1,2) and another point on the circle at (5,3)
3. Endpoints of a diameter at (3,5) and (7,3).
4. Center at (2, -3) and tangent to the x-axis.

Answers:

1. $x^2 + (y-3)^2 = 25$
2. $(x-1)^2 + (y-2)^2 = 17$
3. $(x-5)^2 + (y-4)^2 = 20$
4. $(x-2)^2 + (y+3)^2 = 4$