Factoring Trinomials in the Form of $ax^2 + bx + c$

This kind of trinomial differs from the previous kind we have factored because the coefficient of x is no longer "1".

EXAMPLES of trinomials in the form of $ax^2 + bx + c$:

$6a^2 + 7a - 24$ $\quad a = 6 \quad b = 7 \quad c = -24$

$3p^2 - 16p + 5$ $\quad a = 3 \quad b = -16 \quad c = 5$

$9x^2 + 12x + 4$ $\quad a = 9 \quad b = 12 \quad c = 4$

NOTE that in each case the coefficient of the first term is *not* the number one, nor can we make it "1" by factoring anything out.

In order to factor these trinomials, we must consider the factors of the coefficient of the first term of the trinomial as well as the factors of the <u>third</u> term of the trinomial.

We still use the signs to help us as much as possible.

The following factoring patterns are possible depending upon the signs of the trinomials.

$ax^2 + bx + c = (\square x + \square)(\square x + \square)$

$ax^2 - bx + c = (\square x - \square)(\square x - \square)$

$ax^2 + bx - c = (\square x + \square)(\square x - \square)$

<u>or</u> $(\square x - \square)(\square x + \square)$

$ax^2 - bx - c = (\square x + \square)(\square x - \square)$

<u>or</u> $(\square x - \square)(\square x + \square)$

NOTE that we have two alternative factoring patterns when the signs of the binomials are different. Because we have to consider two different groups of factors, it will make a difference which one is positive and which one is negative.

EXAMPLE: Factor $6a^2 + 7a - 24$

WHAT WE KNOW:

1. The terms of the trinomial do not share a common factor. (Nothing can be factored out.)

2. The third term of the trinomial is negative so the binomials will have different signs (one + and one −).

3. The first term of the trinomial ($6a^2$) comes from the F of FOIL and the third term of the trinomial (−24) comes from the L of FOIL.

4. We must consider all of the factors of 6 and −24.

Possibilities for 6	Possibilities for −24
1, 6	1, −24
2, 3	−1, 24
	2, −12
	−2, 12
	3, −8
	−3, 8
	4, −6
	−4, 6

We must now go through a trial and error process of trying each of the pairs of factors of −24 with each pair of factors of 6. Let's will try $2a$ and $3a$ first:

$$(2a + 1)(3a - 24) \text{ or } (2a - 24)(3a + 1)$$
$$(2a - 1)(3a + 24) \text{ or } (2a + 24)(3a - 1)$$
$$(2a + 2)(3a - 12) \text{ or } (2a - 12)(3a + 2)$$
$$(2a - 2)(3a + 12) \text{ or } (2a + 12)(3a - 2)$$
$$(2a + 2)(3a - 12) \text{ or } (2a - 12)(3a + 2)$$
$$(2a + 3)(3a - 8) \text{ or } (2a - 8)(3a + 3)$$
$$(2a - 3)(3a + 8) \text{ or } (2a + 8)(3a - 3)$$
$$(2a + 4)(3a - 6) \text{ or } (2a - 6)(3a + 4)$$
$$(2a - 4)(3a + 6) \text{ or } (2a + 6)(3a - 4)$$

NOTE that we must be trying each pair of factors in both positions. In the first pairs we had to try −24 with both $2a$ and $3a$ and the +1 with both $2a$ and $3a$. The next step is to check the O and I of FOIL to determine which combination of factors (if any) will give us the correct middle term.

It looks like we have a lot of work to do but we can use one simple fact to help us cut down on the work. If the terms of the trinomial we are factoring have no common factors there will never be any common factors in the correct pair of binomials.

$6a^2 + 7a - 24$ has no common factors.

This means that we can immediately rule out many of the possible combinations.

trial factors		trial factors
$(2a + 1)(3a - 24)$		$(2a - 24)(3a + 1)$
$(2a - 1)(3a + 24)$		$(2a + 24)(3a + 24)$
$(2a + 2)(3a - 12)$		$(2a - 12)(3a + 3)$
$(2a + 3)(3a - 8)$	or or	$(2a + 8)(3a - 3)$
$(2a - 3)(3a + 8)$	or or	$(2a + 8)(3a - 3)$
$(2a + 4)(3a - 6)$	or or	$(2a - 6)(3a + 4)$
$(2a - 4)(3a + 6)$	or	$(2a + 6)(3a - 4)$

By looking for common factors in the binomials, we can quickly rule out all but 2 possibilities:

$(2a + 3)(3a - 8)$ and $(2a - 3)(3a + 8)$

Use the O and I of FOIL to check for the middle term.

$(2a + 3)(3a - 8)$	$(2a - 3)(3a + 8)$
$-16a + 9a = -7a$	$16a - 9a = +7a$ ←The correct sum.

The correct factorization of $6a^2 + 7a - 24$ is $(2a - 3)(3a + 8)$. If none of these combinations had given us the correct middle term, we would have repeated the process using $1a$ and $6a$ in place of $2a$ and $3a$.

NOTE that each possible combination gave us the correct first and third terms ($6a^2$ and -24), but only *one* gave us the correct middle term.

FACTOR $3p^2 - 16p + 5$

WHAT WE KNOW:

1. The third term is positive, so the signs of the binomials will be the same.
2. The middle term is negative, so the signs will both be negative.
3. The trinomial contains no common factors.
4. We must consider factors of 3 and negative factors of 5.

3	5
1, 3	−1, −5

Because 3 and 5 are both prime numbers we do not have any choice of factors.

$(3p - 1)(p - 5)$ or $(3p - 5)(p - 1)$

REMEMBER that −5 and −1 must both be tried with $3p$ and p.

$(3p - 1)(p - 5)$ or $(3p - 5)(p - 1)$

$-15p - 1p = 16p \qquad -3p - 5p = -8p$

Using the O and I of FOIL to check the middle term, we see that the correct factorization is

$(3p - 1)(p - 5)$

FACTOR: $9x^2 + 12x + 4$

WHAT WE KNOW:

1. The third term of the trinomial is positive, so the signs of the binomials will be the same.

2. The middle term is positive, so the signs of the binomials will both be positive.

3. We must consider factors of 9 and 4.

9	4
1, 9	1, 4
3, 3	2, 2

$(3x + 2)(3x + 2)$

$(3x + 1)(3x + 4)$

$(x + 2)(9x + 2)$

$(x + 1)(9x + 4)$ or

$(x + 4)(9x + 1)$

NOTE that in the first three pairs I had only one position to try for each factor. This is because the pairs were the same ($3x$ and $3x$ or 2 and 2).

Use the O and I of FOIL to check each possibility.

Correct → $(3x + 2)(3x + 2)$

$6x + 6x = 12x$

$(3x + 1)(3x + 4)$

$12x + 3x = 15x$

$(x + 2)(9x + 2)$

$2x + 18x = 20x$

$(x + 1)(9x + 4)$

$4x + 9x = 13x$

$(x + 4)(9x + 1)$ x

$+ 36 = 37x$

FACTOR COMPLETELY: $15b^2 - 115b + 70$

WHAT WE KNOW:

1. The terms of the trinomial have a common factor of 5.

2. The third term of the trinomial is positive, so the signs of the binomials will be the same.

3. The middle term of the trinomial is negative, so the signs of the binomials will both be negative.

4. We must factor out a 5.

$5(3b^2 - 23b + 14)$

5. We must consider factors of 3 and negative factors of 14.

3	14
1, 3	−1, −14
	−2, −7

$(b - 2)(3b - 7)$ or $(b - 7)(3b - 2)$

$-7b - 6b = -13b$ $\qquad$ $-2b - 21b = -23b$ ←correct

$(b - 1)(3b - 14)$ or $(b - 14)(3b - 1)$

$-14b - 3b = -17b$ $\qquad$ $-b - 42b = -43b$

Using the O and I of FOIL to check the middle term, we see that the correct factorization is $5(b - 7)(3b - 2)$. The GCF must be in the answer.

FACTOR COMPLETELY: $24x^3y + 14x^2y - 20xy$ WHAT

WE KNOW:

1. The terms of the trinomial have a common factor of $2xy$.

2. The third term of the trinomial is negative, so the signs of the binomials will be different.

3. We must factor out $2xy$.

$$2xy(12x^2 + 7x - 10)$$

4. We must consider factors of 12 and factors of −10.

12	−10
1, 12	1, −10
2, 6	−1, 10
3, 4	2, −5
	−2, 5

By inspecting the pairs of factors carefully, we can see that we can rule out 2 and 6, because no matter which factors of −10 we used, we would have a common factor in one of the binomials.

In every case one binomial has common factors	$(2x - 2)(6x + 5)$	or or	$(2x + 5)(6x - 2)$
	$(2x + 2)(6x - 5)$	or or	$(2x - 5)(6x + 2)$
	$(2x - 1)(6x - 10)$		$(2x + 10)(6x - 1)$
	$(2x + 1)(6x - 10)$		$(2x - 10)(6x + 1)$

Let's try 3 and 4.

$(3x - 2)(4x + 5)$	or	$(3x + 5)(4x - 2)$
$(3x + 2)(4x - 5)$	or	$(3x - 5)(4x + 2)$
$(3x - 1)(4x + 10)$	or	$(3x + 10)(4x - 1)$
$(3x + 1)(4x - 10)$	or	$(3x - 10)(4x + 1)$

The only possibilities here are:

$(3x - 2)(4x + 5)$	$(3x + 2)(4x - 5)$
$(3x + 10)(4x - 1)$	$(3x - 10)(4x + 1)$

The other pairs all have a common factor in one of the binomials.

Checking the middle terms, we see that the correct factorization of $12x^2 + 7 - 10$ is $(3x - 2)(4x + 5)$, and the complete factorization of $24x^3y + 14x^2y - 20xy$ is $2xy(3x - 2)(4x + 5)$. The GCF must be included as part of the answer.

If none of these possibilities had worked, we would go through the same procedure with 1 and 12.

EXERCISES:

Factor Completely:

a. $18x^2 - 9x - 5$

b. $2y^2 + 5y - 12$

c. $24y^2 + 41y + 12$

d. $18y^2 - 39y + 20$

e. $8x^2 - 30x + 25$

f. $16x^2 - 16x - 12$

g. $18x^2 - 27xy + 9y^2$

h. $10x^3 + 12x^2 + 2x$

i. $12x^4y^2 - 17x^3y^3 + 6x^2y^4$

j. $26y^2 + 98y - 24$

KEY:

a. $(3x + 1)(6x - 5)$

b. $(y + 4)(2y - 3)$

c. $(3y + 4)(8y + 3)$

d. $(3y - 4)(6y - 5)$

e. $(2x - 5)(4x - 5)$

f. $4(2x + 1)(2x - 3)$

g. $9(2x - y)(x - y)$

h. $2x(x + 1)(5x + 1)$

i. $x^2y^2(3x - 2y)(4x - 3y)$

j. $2(y + 4)(13y - 3)$