Graphing Linear Equations

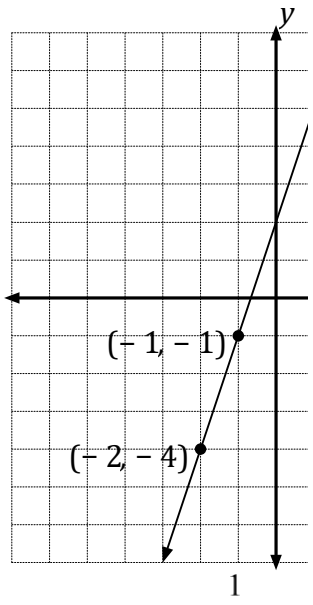
A linear equation has infinitely many ordered pair solutions. The graph of an equation in two variables is a drawing of the ordered pair solutions of the equation. It is not possible to name *all* the solutions. We generally find three ordered pair solutions and graph them. The complete solution set can be shown by drawing a straight line through the graphs of the ordered pairs. An arrow on each end of the line shows that the solution set continues in both directions.

EXAMPLE: Graph $y = 3x + 2$

To find three ordered pair solutions, pick **any** three values for x and solve for y.

Let $x = 1$:	Let $x = -1$:	Let $x = -2$:
$y = 3(1) + 2$	$y = 3(-1) + 2$	$y = 3(-2) + 2$
$y = 3 + 2$	$y = -3 + 2$	$y = -6 + 2$
$y = 5$	$y = -1$	$y = -4$
$(1, 5)$	$(-1, -1)$	$(-2, -4)$

Now we graph the ordered pair solutions $(1, 5)$, $(-1, -1)$, and $(-2, -4)$.



NOTE that the three points fall in a straight line. **EVERY** point on the line is a solution of the equation and can be represented by an ordered pair. Two points are sufficient to draw a straight line, but we generally get a third point as a check.

EXAMPLE: Graph $y = -\frac{1}{4}x + 1$

NOTE that the coefficient of x in this equation is a fraction. When this occurs we want to pick values for x which will allow us to eliminate the fraction. As the denominator of the fraction is 4, the easiest choices to work with will be multiples of 4, such as 0, 4 and -4.

Let $x = 0$:

$y = -\frac{1}{4}(0) + 1$

$y = 0 + 1$

$y = 1$

$(0, 1)$

Let $x = 4$:

$y = -\frac{1}{4}(4) + 1$

$y = -1 + 1$

$y = 0$

$(4, 0)$

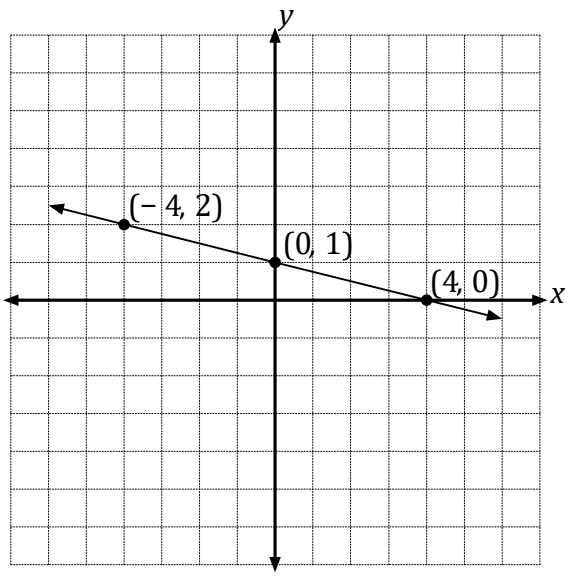
Let $x = -4$:

$y = -\frac{1}{4}(-4) + 1$

$y = 1 + 1$

$y = 2$

$(-4, 2)$



Any time the coefficient of x is a fraction, convenient choices for x are zero, the denominator, and the opposite of the denominator.

$y = -\frac{2}{3}x + 8$ choose 0, 3, and −3

$y = -\frac{2}{5}x + 4$ choose 0, 5, and −5

Sometimes the equation is in the form of Ax + By = C, and in this case we can solve the equation for y first.

EXAMPLE: Graph $2x + 3y = 12$

To solve the equation for y, follow these steps:

$2x + 3y = 12$

1. To isolate the y term, add the opposite of the term containing x to both sides of the equation. $2x - 2x + 3y = -2x + 12$

$3y = -2x + 12$

2. Divide both sides of the equation by the coefficient of y. $3y = -2x + 12$ This means **both terms** on the right-hand side must be divided by the coefficient. $\frac{3}{3}y = -\frac{2}{3}x + \frac{12}{3}$

$2y = -\frac{2}{3}x + 4$

Once the equation is in the form of $y = mx + b$, the ordered pair solutions can be found by picking values for x and solving for y. As the coefficient of x is $-\frac{2}{3}$ we would pick 0, 3 and -3 to get the ordered pairs.

Let $x = 0$:

$y = -\frac{2}{3}(0) + 4$

$y = +0 4$

$y = 4$

(0, 4)

Let $x = 3$:

$y = -\frac{2}{3}(3) + 4$

$y = -+2 4$

$y = 2$

(3, 2)

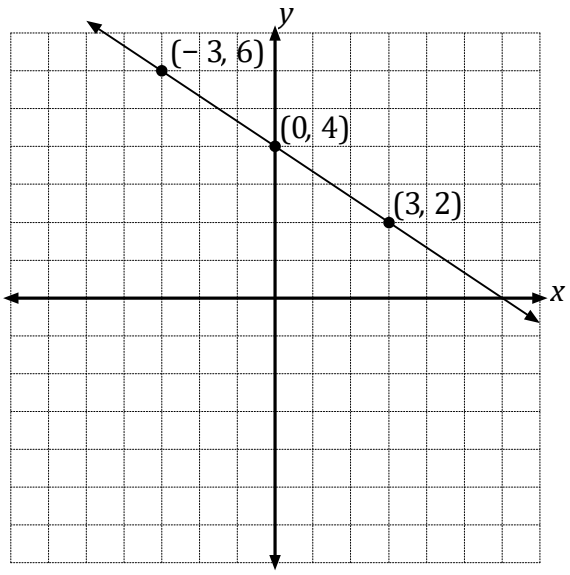
Let $x = -3$:

$y = --+\frac{2}{3}(-3) 4$

$y = +2 4$

$y = 6$

(−3, 6)



Now we will practice rewriting equations in the form $Ax + By = C$ to their equivalent $y = mx + b$ form.

EXAMPLE: Solve for y: $x + 4y = -6$

$x + - + = - -(x)$ Add $-x$ to both sides

$4y \quad x$

6

Divide each term by 4

$4y = - -x6$

$4 \quad x$

$6 \_ y$

$= - - \_$

$4 \quad 4 \quad 4$

$1 \quad 3$

$y = - x - \ \_ \quad \_$

$4 \quad 2$

EXAMPLE: Solve for y: $-2x - 4y = 8$

$- + - = +2x2x\ 4y$

$2x8$

$- = +4y\ 2x$ Add $2x$ to both sides

8

$-4\ 2x\ 8\frac{y}{} = +$ Divide each term by -4

$-4\ \underline{-4}\ -4\ \underline{y} = - \_\_$

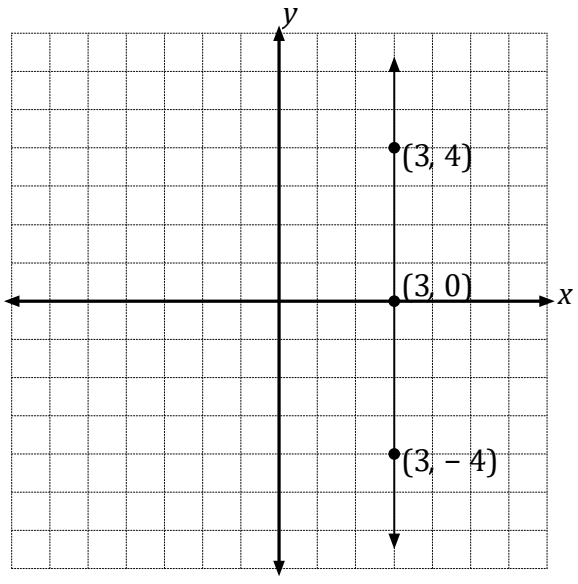
$\frac{1}{2}$

$x\ -2$

EXAMPLE: Graph the equation $x = 3$.

NOTICE that the equation $x = 3$ does not mention y. This equation could be written as $0 \cdot y\ x + = 3$. In this case no matter what value y has, because y is multiplied by 0, x will always be 3. This graph will be a **vertical line** through the point where $x = 3$.

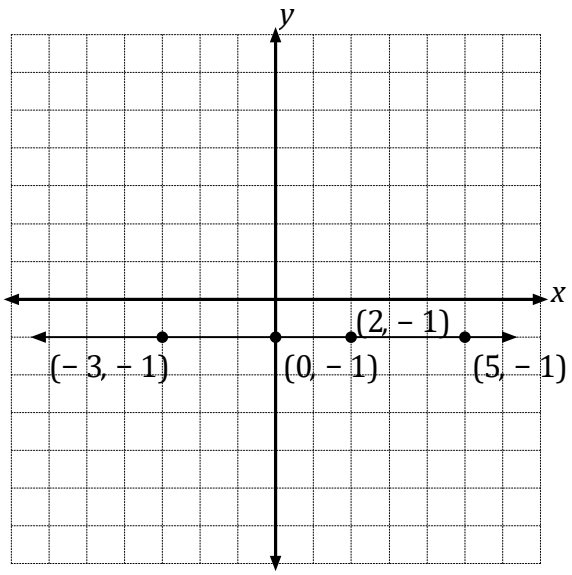
x	y
3	-4
3	-2
3	0
3	2
3	4



EXAMPLE: Graph the equation $y = -1$.

NOTICE that this equation does not mention x. This equation could be written as $0 \cdot x + y = -1$. In this case no matter what value x has, because x is multiplied by 0, y will always be -1. This graph will be a **horizontal line** through the point where $y = -1$.

x	y
-3	-1
-1	-1
0	-1
2	-1
5	-1



PRACTICE

Graph the lines of the following equations. Find at least three ordered pairs associated with each line.

a. $3x - y = 4$

b. $2x + y = 1$

c. $y = \frac{1}{2}x + 3$

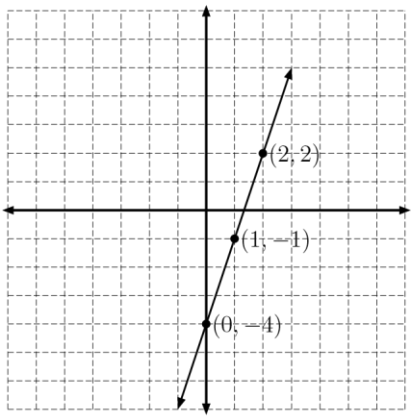
d. $y = -\frac{1}{2}x + 3$

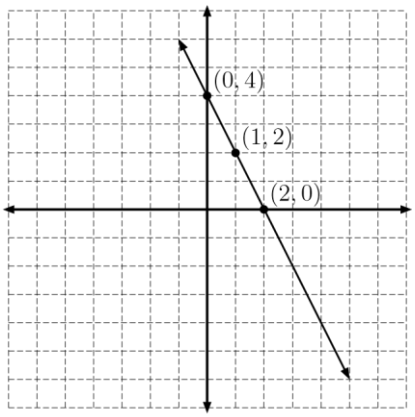
e. $2x + y = 4$

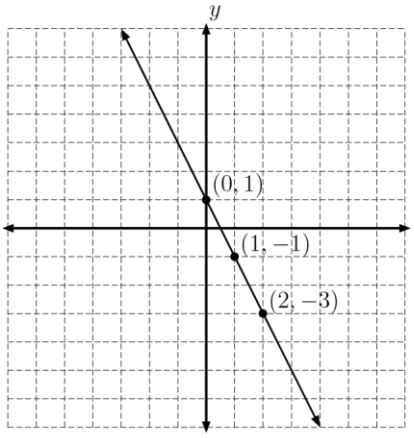
f. $3x + 4y = 12$

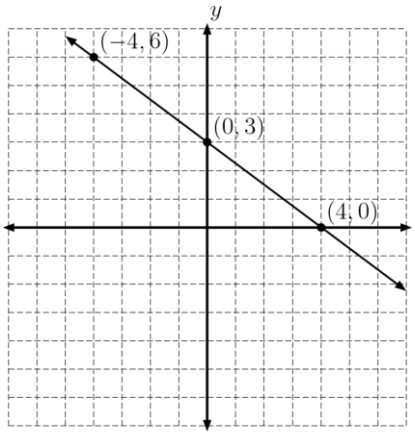
g. $x = -4$

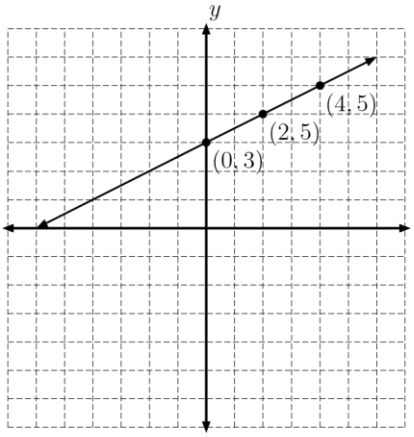
h. $y = 3$

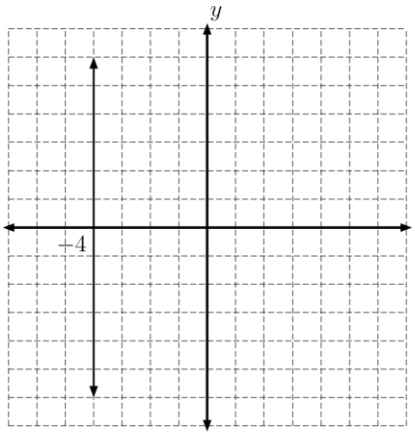


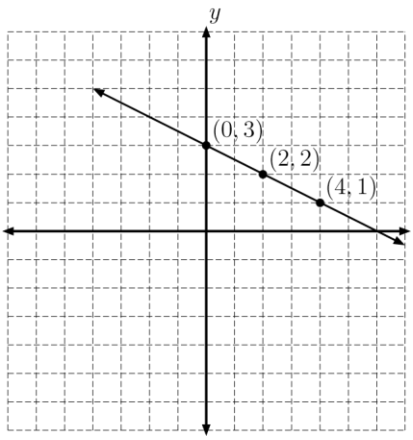


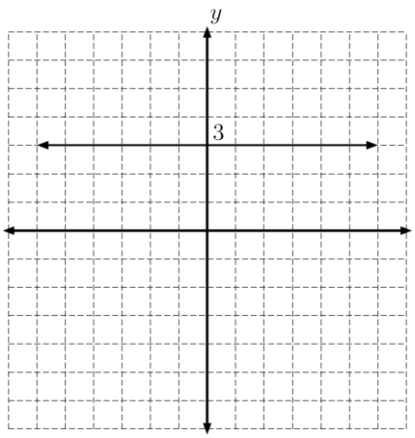












a. e.

xx

b.f.

xx

c.g.

xx

d.h.

xx