MAC 1105 Solving Quadratic Equations by Completing the Square

Quadratic Equation – Any equation which can be written in the form of $ax^2 + bx + c = 0$.

Completing the Square – Solving a Quadratic Equation by creating a Polynomial which can be factored as a Perfect Square Trinomial.

Example: Solve $x^2 + 2x - 8 = 0$ by Completing the Square.

a **b** **c**

Step 1: Move c to the opposite side of the equation.

$x^2 + 2x - 8 = 0 \rightarrow \mathbf{x^2 + 2x = 8}$

$+8 \ +8$

b 2

Step 2: Add ( ) to each side of the equation. **b = 2**

2

$(b)^2 = (2 \quad 2 \quad 2 = 1 \rightarrow x^2 + 2x + \mathbf{1} = 8 + \mathbf{1} \rightarrow \mathbf{x^2 + 2x + 1 = 9}$

$) \ = 1$

2 2

Step 3: Factor $x^2 + 2x + 1$ as a Perfect Square Trinomial. (Remember $\sqrt{\ }$ and squares are inverses.) $x^2 +$

$2x + 1 = 9$

$(x + 1)(x + 1) = 9$

$\mathbf{(x + 1)^2 = 9}$

Step 4: Square Root both sides of the equation.

$\sqrt{(x + 1)^2} = \pm\sqrt{9} \rightarrow \mathbf{x + 1 = \pm 3}$

Step 5: Solve for x.

$x + 1 = 3$ $\qquad$ $x + 1 = -3$

$-1 \ -1$ $\qquad$ $-1 \ \ -1$

$\mathbf{x = 2}$ $\qquad$ $\mathbf{x = -4}$

Example: Solve $2x^2 - 5x - 3 = 0$ by Completing the Square.

Step 1: Use Algebra to move c to the opposite side of the equation.

$2x^2 - 5x - 3 = 0 \rightarrow 2x^2 - 5x = 3$

$+3 + 3$

Step 2: Divide both sides of the equation by a. (You want the leading coefficient to be 1.)

$\frac{2}{2} - \frac{5}{2}x + \frac{3}{2} = \frac{0}{2}$

Step 3: Add $\left(\frac{b}{2}\right)^2$ to each side of the equation.

$\left(\frac{-5}{2}\right)^2 = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}$ $\rightarrow$ $x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$ $\rightarrow$ $x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{24}{16} + \frac{25}{16}$ $\rightarrow$ $\mathbf{x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{49}{16}}$

Step 4: Factor $x^2 - \frac{5}{2}x + \frac{25}{16}$ as a Perfect Square Trinomial.

$x^2 - \frac{5}{2}x + \frac{25}{16} = (x - \frac{5}{4})^2$ $\rightarrow$ $\mathbf{(x - \frac{5}{4})^2 = \frac{49}{16}}$

Step 5: Square Root both sides of the equation.

$\sqrt{(x - \frac{5}{4})^2} = \pm\sqrt{\frac{49}{16}}$ $\rightarrow$ $\mathbf{x - \frac{5}{4} = \pm\frac{7}{4}}$

Step 6: Solve for x.

$x - \frac{5}{4} = \frac{7}{4}$

$+\frac{5}{4} + \frac{5}{4}$

$\mathbf{x = 3}$

$x - \frac{5}{4} = \frac{-7}{4}$

$+\frac{5}{4} + \frac{5}{4}$

$\mathbf{x = -\frac{1}{2}}$

Practice Problems:

Solve the following Quadratic Equations by Completing the Square:

1) $x^2 + 4x - 21 = 0$ Solution: $x = \mathbf{3}, \mathbf{-7}$

2) $x^2 - 12x = -20$ Solution: $x = \mathbf{2}, \mathbf{10}$ 3) $3x^2 - 5x + 2 = 0$ Solution: $x = \frac{2}{3}, \mathbf{1}$

4) $4x^2 + 19x = -12$ Solution: $x = -\frac{3}{4}, \mathbf{-4}$