MAC 1105 Solving Quadratic Equations by Completing the Square

Quadratic Equation – Any equation which can be written in the form of $ax^2 + bx + c = 0$.

Completing the Square – Solving a Quadratic Equation by creating a Polynomial which can be factored as a Perfect Square Trinomial.

Example: Solve $x^2 + 2x - 8 = 0$ by Completing the Square.

a **b c**

Step 1: Move c to the opposite side of the equation.

 $x^{2} + 2x - 8 = 0 \rightarrow x^{2} + 2x = 8$ + 8 + 8

b 2 Step 2: Add () to each side of the equation. **b** = **2**

2

 $(^{b})^{2} = (^{2} \quad ^{2} \quad ^{2} = 1 \qquad \rightarrow \quad x^{2} + 2x + 1 = 8 + 1 \quad \rightarrow \quad x^{2} + 2x + 1 = 9$) = 1 2 2

Step 3: Factor $x^2 + 2x + 1$ as a Perfect Square Trinomial. (Remember \sqrt{and} squares are inverses.) $x^2 + 3x^2 + 3x^$

2x + 1 = 9

(x+1)(x+1) = 9

 $(x + 1)^2 = 9$

Step 4: Square Root both sides of the equation.

 $\sqrt{(x+1)^2} = \pm \sqrt{9} \rightarrow x+1 = \pm 3$

Step 5: Solve for x.

x + 1 = 3	x + 1 = -3
-1 -1	-1 -1
x = 2	x = -4

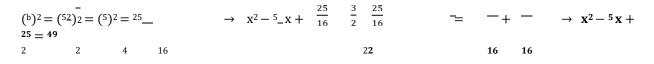
Example: Solve $2x^2 - 5x - 3 = 0$ by Completing the Square.

Step 1: Use Algebra to move c to the opposite side of the equation.

 $2x^2 - 5x - 3 = 0 \rightarrow 2x^2 - 5x = 3$ + 3 + 3

Step 2: Divide both sides of the equation by a. (You want the leading coefficient to be 1.)

$$\frac{-}{x} = \frac{5}{2} = \frac{3}{2} = 2 = 2 = 2$$
Step 3: Add () to each side of the equation.



Step 4: Factor x^{2} 5_{x} $\frac{25}{16}$ + as a Perfect Square Trinomial.

$$x^2 - 5_x = \frac{25}{16} + = (x - 5)^2 \rightarrow (x - 5)^2 = \frac{49}{16}$$

Step 5: Square Root both sides of the equation.

$$\sqrt[5]{(x-1)^2 = \pm \sqrt{\frac{1}{16}}} \rightarrow x - \frac{5}{4} = \pm \frac{7}{4}$$

Step 6: Solve for x.

$x - \frac{5}{4} = \frac{7}{4}$	$x - \frac{5}{4} = \frac{-7}{4}$
$+\frac{5}{4}+\frac{5}{4}$	$+\frac{5}{4}+\frac{5}{4}$
x = 3	$x = -\frac{1}{2}$

Practice Problems:

Solve the following Quadratic Equations by Completing the Square:

1) $x^{2} + 4x - 21 = 0$ 2) $x^{2} - 12x = -20$ Solution: x = 2, **10** 3) $3x^{2} - 5x + 2 = 0$ Solution: $x = {}^{2}$, **1** 3 4) $4x^{2} + 19x = -12$ Solution: $x = {}^{3}$, -44