

Simplifying Rational Expressions

A rational expression is a quotient of two monomials and/or polynomials.

$$\frac{3x+1}{x^2-1} \cdot \frac{3y}{1}$$

Examples: $\frac{2}{5x^2+2x}$, $\frac{5y^2}{2}$, etc.

Beginning Steps:

- Factor out the greatest common factor (GCF), if any, in all numerators and denominators.
- Factor completely** all numerators and denominators.
- Remaining steps depend on the type of problem. See examples below for three types of problems: (I) single expression; (II) products and quotients; (III) sums and differences.

Type I: Single Rational Expression

Example #1 - Simplify: $\frac{w^2 + w - 12}{w^2 + 8w + 16}$

Solution

numerator or denominator

completely

1. there is no GCF in either

2. factor numerator and denominator

3. cancel any like factors (representing 1)

$$\frac{w^2 + w - 12}{w^2 + 8w + 16} = \frac{(w-3)(w+4)}{(w+4)(w+4)} = \frac{(w-3)}{(w+4)}$$

Example #2 - Simplify: $\frac{3x+6}{3x}$

Solution

1. factor out GCF (of 3) in the numerator

2. there's no other factorization to be done

3. cancel like factors (representing 1)

$$\frac{3x+6}{3x} = \frac{3(x+2)}{3x} = \frac{x+2}{x}$$

Note

it is **incorrect** to cancel as follows: $\frac{3x+6}{3x} \neq 7$ since the 3x in 3x+6 is not a factor but a term of the sum: 3x+6. **3x**

However, $\frac{3(x+2)}{3x} = \frac{x+2}{x}$ is correct.

Example #3 - Simplify: $\frac{150x - 6x^3}{6x^2 - 27x - 15}$

Solution

1. factor out GCF of 6x in numerator and of 3 denominator

2. factor numerator and denominator completely

$$\frac{150x - 6x^3}{6x^2 - 27x - 15} = \frac{6x(25 - x^2)}{3(2x^2 - 9x - 5)} = \frac{3 \cdot 2x(5+x)(5-x)}{3 \cdot 2x(5+x)(5-x)} = \frac{x(5-x)}{5+x}$$

$$3(2x + 1)(x - 5)$$

$$3 \cdot 2x(5 + x) \cdot (-1)(x - 5)$$

$$= 3.$$

$$3(2x + 1)(x - 5)$$

cancel like factors (representing 1)

$$= \frac{-2x(5 + x)}{2x + 1}$$

Note : $5 - x = -1(-5 + x) = -1(x - 5)$. (factoring out -1 and rewriting)

Type II: Products and Quotients

2 2

Example #4 -

Simplify: $4 \frac{x^2 + 1x - 9}{x + 1} \div 10 \frac{x^2 + 8x + 7}{2x - 3} + 19x + 6$

• $52x + x - 103$.

$$4x^2 - 9 \quad 10x^2 + 19x + 6 \quad \frac{5x + 10}{x + 1}$$

+ 7 • $2x - 3$

$$= \frac{4x^2 - 9}{x + 1} \cdot 10 \frac{x^2 + 8x + 7}{2x - 3} \cdot \frac{5x + 10}{x + 1}$$

$$\frac{\cancel{(2x + 3)}(2x - 3)}{x + 1} \cdot \frac{(x + 7)(x + 1)}{(2x + 3)(5x + 2)} \cdot \frac{5(x + 2)}{2x - 3}$$

=(iii) $\frac{5(x + 2)(x + 7)}{5x + 2}$ write final answer as one fraction

Solution -

$$x + 1 \div x^2 + 8x$$

(i) change division to mult. by the reciprocal

(ii) factor completely and cancel

Example #5 -

Simplify: $\frac{2x^2 - 9x - 5}{3x - 1} \cdot \frac{6x - 2}{x^2 + 2x - 8}$.

Solution

$$\frac{2x^2 - 9x - 5}{3x - 1} \cdot \frac{6x - 2}{x^2 + 2x - 8}$$

$$= \frac{\cancel{3x - 1}}{(2x + 1)(x - 5)} \cdot \frac{\cancel{2}(x + 4)(x - 2)}{2(3x - 1)}$$

(i) factor completely

=(ii) $\frac{(x + 4)(x - 2)}{2(2x + 1)(x - 5)}$ cancel like factors

Type III: Sums and Differences

Example #6 -

Add and simplify:

$$\frac{y^2 + 12y + 20}{3y} + \frac{y^2 + 7 - 10}{(y + 10)(y - 10)}$$

(i) factor denominators & find LCD

Solution

(ii) LCD is $(y + 10)(y + 2)(y - 10)$

$$= \frac{y^2 + 12y + 20}{(y + 10)(y + 2)} + \frac{y^2 + 7 - 10}{(y + 10)(y - 10)}$$

$$= \frac{3y}{(y + 10)(y + 2)} \cdot \frac{(y - 10)}{(y - 10)} + \frac{y + 7}{(y + 10)(y - 10)} \cdot \frac{(y + 2)}{(y + 2)}$$

(iii) write all fractions with LCD

$$= \frac{3y^2 - 30y}{(y + 10)(y + 2)(y - 10)} + \frac{y^2 + 9y + 14}{(y + 10)(y + 2)(y - 10)}$$

(iv) add fractions

$$= \frac{-(y + 10)(y + 2)(y - 10)}{(y + 10)(y + 2)(y - 10)}$$

$$3y^2 - 30y + y^2 + 9y + 14$$

=(v) if possible, factor and reduce

$$(y + 10)(y + 2)(y - 10)$$

=(vi)

$4y^2 - 21y + 14$
$(y + 10)(y + 2)(y - 10)$

(cannot factor or reduce further)

Example #7 -

Subtract and simplify:

$$\frac{x + 3}{x + 3} - \frac{5}{5}$$

$$\frac{x + 3}{2x^2 + 13x + 6} - \frac{5}{6x + 3}$$

Solution

$$2x^2 + 13x + 6$$

6x + 3(i) factor all denominators and find LCD

$$= \frac{x + 3}{(2x + 1)(x + 6)} - \frac{5}{3(2x + 1)(x + 6)}$$

(ii) LCD is: $3(2x + 1)(x + 6)$

$$= \frac{(x + 3) \cdot 3}{3(2x + 1)(x + 6)} - \frac{5}{3(2x + 1)(x + 6)}$$

(iii) write all fractions with LCD

$$= \frac{3(x + 3) - 5}{3(2x + 1)(x + 6)}$$

(iv) subtract fractions

$$= \frac{3(2x + 1)(x + 6) - 5(2x + 1)(x + 6)}{3(2x + 1)(x + 6)}$$

$$= \frac{3x + 9 - 5x - 30}{3(2x + 1)(x + 6)} = \frac{-2x - 21}{3(2x + 1)(x + 6)}$$

=(v) (cannot factor or reduce further)