Slope-Intercept Form and Point-Slope Form

Slope of the line	$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form	$y = mx + b$ m *is slope;* b *is y-intercept*
Point-Slope Form	$y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$
Slope of parallel lines	$m_1 = m_2$ (slopes are the same)
Slope of perpendicular lines	$m_1 m_2 = -1$ (slopes are opposite & reciprocal)
Equations of Horizontal and Vertical Lines	$y = b$ horizontal line $x = a$ vertical line, where a & b are constants

Example (1): Write the slope - intercept equation of a line which passes through (0,-7) whose slope is 2.

Solution:

Slope-intercept equation is $y = mx + b$ **.** What we need to complete this equation are slope (m) & y-intercept $(0,b)$, and the problem provides both information.

$m = 2$ and $b = -7$ The equation of the line is $y = 2x - 7$

Example (2): Write the slope-intercept equation of a line which passes through $(0,4)$ and $(3,-5)$.

x_1, y_1

x_2, y_2

Solution:

Slope-intercept equation is $y = mx + b$ **.** What we need to complete this equation are slope (m) & y-intercept $(0,b)$, however, we only have y-intercept.

To find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 4}{3 - 0} = \frac{-9}{3} = -3$

$m = -3$ and $b = 4$ The equation of the line is $y = -3x + 4$

x_1, y_1

Example (3): Write the slope-intercept equation of a line which passes through $(-1,4)$ whose slope is 5.

Solution:

Slope-intercept equation is $y = mx + b$ **.** What we need to complete this equation are slope (m) & y-intercept $(0,b)$, however, we only have slope. Here there are two ways to find the equation of the line.

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 5$, $(x_1, y_1) = (-1,4)$ $\quad 4 = 5(-1) + b$

$\Rightarrow \quad b = 9$ The

equation of the line is $y = 5x + 9$

Method II Since we are given slope m and an ordered pair (x_1, y_1), we can use Point-slope form to find equation of the line.

Point-slope form is $y - y_1 = m(x - x_1)$ $\quad m = 5$, $(x_1, y_1) = (-1,4)$ $\Rightarrow$ $y - 4 = 5(x + 1)$ Simplify the parenthesis

$\Rightarrow \quad y = 5x + 5 + 4$ Distribute 5 into parenthesis

$$\Rightarrow \quad y = +5\ 9x$$

$x_1,\qquad y_1$

Example (4): Write the slope-intercept equation of a line which passes through $(1,3)$and x_2, y_2

$(- -5, 1)$.

Solution:

Slope-intercept equation is $y = +mx\ b$ **.** What we need to complete this equation are slope $( )m$ & y-intercept $(0,b)$. However, we are given two ordered pairs$(x\ y_1,\ _1)$ and $(x\ y_2,\ _2)$ without slope and yintercept. Therefore, we need to find the slope first. Then we can use the two methods discussed on Example (3) to find the equation of the line.

$$(x\ y_1,\ _1) \qquad (x\ y_2,\ _2)$$

To find the slope between two ordered pairs, $(1,3)$ and $(- -5, 1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{- -1\ 3}{- -5\ 1} = \frac{-4}{-6} = \frac{2}{3}$

Method I Now we have slope, we will substitute m and $(x\ y_1,\ _1)$ in the form $y = +mx\ b$ to solve for b.

$m = \frac{2}{3}$, $(x\ y_1,\ _1) = (1,3)$ $\qquad 3 = \frac{2}{3}( )1 + b$

$$\Rightarrow \quad b = \frac{7}{3}$$

The equation of the line is $y = \frac{2}{3}x + \frac{7}{3}$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y\ m\ x\ x = (- +_1)\ y_1$ $\qquad y = \frac{2}{3}(x - +1\ 3)$

$\frac{2}{3}$ $m =$, $(x\ y_1,\ _1) = (1,3)$ $\qquad$ $\frac{2}{3}$

$$\Rightarrow \quad y = \frac{2}{3}x - + \frac{2}{3}\ 3$$ Distribute into parenthesis

$$\Rightarrow \quad y = \frac{2}{3}x - \frac{2}{3} + \frac{9}{3} \qquad \text{Combine like term}$$

$$\Rightarrow \quad y = \frac{2}{3}x + \frac{7}{3}$$

Example (5): Write the slope-intercept equation of a line which is parallel to $y = -4\ 2x$, passing through $(1,3)$.

x_1, y_1 **Solution:**

Slope-intercept equation is $y = +mx\ b$ **.** What we need to complete this equation are slope $(\ )m$ & y-intercept $(0,b)$. Since the line we're looking for is **parallel** to $y = -4\ 2x$ **, their slopes are the same**, $m=4$.

Method I We will substitute m and $(x\ y_1,\ _1)$ in the form $y = +mx\ b$ to solve for b.

$m=4,\ (x\ y_1,\ _1)=(1,3) \qquad 3 = 4\ 1(\ ) + b \ \Rightarrow \ b = -1$

The equation of the line is $y = -4\ 1x$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y\ m\ x\ x = (\ -\ +_1)\ y_1 \quad y = 4(x - +1)\ 3$

$m=4,\ (x\ y_1,\ _1)=(1,3) \qquad \Rightarrow \qquad y = -\ +4\ 4\ 3x$ Distribute 4 into parenthesis

$\Rightarrow \ y = -4\ 1x$ Combine like term

Example (6): Write the slope-intercept equation of a line which is perpendicular to $\frac{1}{3}$ x^1, y^1 $y = -x + 4$, passing through $(-3,5)$.

Solution:

Slope-intercept equation is $y = +mx\ b$ **.** What we need to complete this equation are the slope $( )m$ & y-intercept $(0,b)$. Since our line is **perpendicular** to $y = - + \frac{1}{3}x\ 4$ **(**which was given**),** we can find the slope of our line by **taking the opposite sign and using the reciprocal of the given line** which has a slope of $m = -\frac{1}{3}$. Therefore, **the slope of our line is** $m = 3$ (the perpendicular one to the given line)

Method I We will substitute m and $(x\ y_{1},\ _{1})$ in the form $y = +mx\ b$ to solve for b.

$m = 3,\ (x\ y_{1},\ _{1}) = (-3,5)\ 5 = - +3(3)\ b$

$\Rightarrow \quad 5 = - +9\ b$

$\Rightarrow \quad b = 14$ The

equation of the line is $y = +3\ 14x$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y\ m\ x\ x = (- +_{1})\ y_{1}\ y = 3(x - - +(3))\ 5 \quad m = 3,\ (x\ y_{1},\ _{1}) = (-3,5)$

$\Rightarrow \quad y = 3(x + +3)\ 5$ Simplify the parenthesis

$\Rightarrow \quad y = + +3\ 9\ 5x$ Distribute 5 into parenthesis

$\Rightarrow \quad y = +3\ 14x$

$x_{1},\ y_{1}$ **Example (7):** Write an equation of a vertical line which passes through $(1,6)$.

Solution:

The equation of a vertical line is $x\ a$

The x-coordinate of the point $(1,6)$ is 1. Therefore, the equation of the vertical line is $x = 1$

$x_{1},\ y_{1}$

Example (8): Write an equation of a horizontal line which passes through $\left(-\frac{3}{},-\frac{5}{}\right)$.

□ 4 6□

Solution:

The equation of a horizontal line is $y\ b$□

□ 3,□ 5□□ $\frac{5}{6}$ is □. Therefore, the equation of the horizontal line is $\frac{5}{6}$ y□□ The y-coordinate of the point □

□ 4 6□

Exercises:

1. Write the slope - intercept equation of a line which passes through (0,5) whose slope is 4.
2. Write the slope-intercept equation of a line which passes through □0,□3□ and □4,5□.
3. Write the slope-intercept equation of a line which passes through □4,0□and □7,□1□.
4. Write the slope-intercept equation of a line which is parallel to y□ □3 5x , passing through □□6,3□
5. Write the slope-intercept equation of a line which is perpendicular to y□ □7 2x , passing through □3,2□
6. Write an equation of a horizontal line which passes through □5,□1□
7. Write an equation of a vertical line which passes through □□8, _ 7 □□ .

□ 3□

Answers:

1. y□ □4 5x **2.** y□ □2 3x **3.** y□□ □$\frac{1}{3}x\ \frac{4}{3}$ **4.** y□ □3x 21 **5.** y□□ □$\frac{1}{7}x\ \frac{17}{7}$

6. y□□1 **7.** x□8