Solving Linear Applications

Many real-life applications can be approximated by a linear model. Information can come in a variety of ways to enable us to create a model. These examples show some of the ways that we can build our model.
Our basic equations that we can use are:

$\boldsymbol{y = mx + b}$ $\quad\quad$ $\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad\quad$ $\boldsymbol{y - y_1 = m(x - x_1)}$

Example 1: Given both m and b.

1. A student buys a basic phone that has a base charge of $16.44 per month, with each minute of calling time costing them $.20. Develop a model that can help predict the cost each month. We need 2 things for the linear model: slope and y-intercept (base/fixed costs). a. What we know:
   a. The model is $C(x) = mx + b$.
   b. The base cost is $16.44, so b = 16.44.
   c. The slope or the part that changes is $.20 per minute. The number of minutes of calls will be x. This gives m = .20.

   Thus, our cost equation is $\boldsymbol{C(x) = .20x + 16.44}$

   b. If they know that they only have $66.44 this month for the phone bill, how many minutes can they use?

   If C(x) = 66.44, we need to solve this for x:

$$66.44 = .20(x) + 16.44$$
$$66.44 = .20(x) + 16.44$$
$$50.00 = .20x$$
$$\frac{50.00}{.20} = \frac{.20x}{.20}$$
$$x = 250$$

Thus, the student can use their phone for 250 minutes this month and stay on budget!

Example 2: Given b and a point.

2. The population of a small town in the panhandle of Florida has been showing an incline in the years from 2002 to 2023. In 2002, the population was 15,200. In 2023, a survey shows that the population has grown to 16,460.

   Write a linear equation modelling the population of this town, P(t), as a function of t, the number of years since 2002.

   a. What we know:

   a. The model is $P(t) = mt + b$.
   b. Given: (2002, 15, 200) and (2023, 16, 460).
   c. The base population in 2002 is 15, 200 so b = 15, 200.
   d. The slope or the part that changes is calculated by using the two points we were given.

$$m = \frac{y_2 - y_1}{\quad} \quad \frac{16{,}460 - 15{,}200}{\quad} \quad \frac{1260}{\quad} = \quad = \quad = 60$$

$x_2 - x_1$ $\quad 2023 - 2002 \quad 21$

Now we know that the small town is gaining 60 people per year.

Thus, our population equation is $\boldsymbol{P(t) = 60t + 15,200}$

b. Use your equation to predict the population in the year 2040 to help understand the need for more teachers, police officers, etc.

a. We know the model: $P(t) = 60t + 15{,}200$

b. To calculate t, the time since 2002, we take the given year and subtract: 2040 - 2002= 38 c. Substitute this into our model.

$$\boldsymbol{P(38)} = 60(38) + 15{,}200$$

$= 17{,}480$. This is the predicted population in the year 2040.

Example 3: Given 2 Random Points

3. A company was doing a study on their advertising budget and its affect on sales. They noted that when they spent $40,000 on advertising, they sold 100,000 bags of candy. They also noted that when they spent $60,000, they sold 200,000 bags of candy. They would like to develop a linear model that will give them the correlation between advertising budget and sales.

a. We can pair the given information as two ordered pairs; ($advertising, sales). This give us two points to work with. (40,000, 100,000) and (60,000, 200,000).

b. The model is $C(x) = mx + b$. We need to determine the slope (m) and the y-intercept (b).

c. $m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{200{,}000 - 100{,}000}{60{,}000 - 40{,}000} = \frac{100{,}000}{20{,}000} == 5$, which we can interpret as 5 bags sold per dollar spent in advertising.

d. We can then calculate a b(y-intercept) for this model. We can use: $y - y_1 = m(x - x_1)$ $y - 100{,}000 = 5(x - 40{,}000)$; using the point $(40{,}000, 100{,}000)$ $y - 100{,}000 = 5x - 200{,}000$; then solve this for y.

$$y = 5x - 300{,}000$$

So our linear model is $C(x) = 5x - 300{,}000$.

e. If the company wants to sell 350,000 bags of candy, how much should they spend in advertising?

$C(x) = 5x - 300{,}000$ is the model.

$350{,}000 = 5x - 300{,}000$;we want $C(x) = 350{,}000$

$x = 130{,}000$.

To interpret, the company should spend $130,000 on advertising to reach their target goal of selling 350,000 bags of candy.

You Try:

A company sells laptops. They are trying to develop a model that will show them how many laptops will be sold if they spend a certain amount on marketing. They have data that shows that if they spend $150,000, they will sell 65,000 laptops. If they spend $175,000, they will sell 77,500 laptops. Calculate the linear model that shows the correlation between marketing and estimated laptops to be sold. ANS: $M(x) = .5x - 10{,}000$.