Summary of Integration by Substitution

Steps to integration by substitution:

Example 1: Consider $\int (x^2 + 1)\, 2x\, dx$

1) Let u equal the expression inside the parenthesis.
   Solution: **u =** $\boldsymbol{x^2 + 1}$

2) Find du. Solution: **du = 2x dx**

3) Substitute. Solution: $\int (\boldsymbol{x^2 + 1}) \boldsymbol{2x\, dx} = \int \mathbf{u\, du}$

   ↑ ↑
   u **du**

4) Take the antiderivative of u. Solution: $\frac{u2}{2} + c$

5) Substitute $x^2 + 1$ back in for u.

Final Solution: $\frac{(\boldsymbol{x^2+1})^2}{\mathbf{2}} + \boldsymbol{c}$

Example 2: Consider $\int 3x\, e^{x2} dx$

1) Let u equal the expression inside the exponent. Solution: **u =** $\boldsymbol{x^2}$

2) Find du. Solution: **du = 2x dx**

3) We need to be able to substitute something in for 3x dx. But du = 2x dx. So use algebra to get the right side of #2 to equal 3x dx.

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$\frac{3}{2} \quad = \frac{3}{2} * 2 \quad dux\, dx$ so $\boldsymbol{du = 3x\, dx}$

2

4) Substitute. Solution: $\int 3x\,dx\, e^{x^2} = \int \frac{3}{2} du\, e^u = \frac{3}{2}\int du\, e^u$

↑ ↑

$\frac{3}{2}du \quad e^u$

5) Take the antiderivative of e^u. Solution: $\frac{3}{2}e^u + C$

6) Substitute x^2 back in for u.

Final Solution: $\frac{3}{2}e^{x^2} + C$

Example 3: Consider $\int 4x \sin(x^2)\, dx$

1) Let u equal x^2. Solution: **u =** x^2

2) Find du. Solution: **du = 2x dx**

3) We need to be able to substitute something in for 4x dx. But du = 2x dx. So use algebra to get the right side of #2 to equal 4x dx.

$2\text{ du} = 2 * 2\text{x dx}$ so $\mathbf{2\ du = 4x\ dx}$ 4)

Substitute.

Solution: $\int \mathbf{4x\ dx\ sin(x^2)} = \int \mathbf{sin(u) 2du} = \mathbf{= 2\int du\ sin\ (u)}$

↑ ↑

2 du sin(u)

5) Take the antiderivative of sin(u). Solution: $2(-cos(u)) = -2cos(u) + C$

6) Substitute x^2 back in for u.

Final Solution: $-2\cos(x^2) + C$

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Example 4: Consider $\int \frac{2}{x}\ln(x^2)\,dx$

1) Let u equal $\ln(x^2)$. Solution: **u =** $\mathbf{\ln(x^2)}$

2) Find du. Solution: **du =** $\frac{1}{x^2}\,2x\,dx = \frac{2}{x}\,dx$

3) We need to be able to substitute something in for $\frac{2}{x}$ dx. But du = $\frac{2}{x}$ dx.

$$du = \frac{2}{x}\,dx$$

4) Substitute.

Solution: $\int \frac{2}{x}dx\,\ln(x^2) = \int u\,du$

↑ ↑

du **u**

5) Take the antiderivative of u.

Solution: $\frac{u^2}{2} + C$

6) Substitute $\ln(x^2)$ back in for u.

Final Solution: $\frac{(\ln(x^2))^2}{2} + C$

Practice Problems:

Find the Indefinite Integral using substitution:

1) $\int 5x(1 + x^2)^3\, dx$

2) $\int -x^3(2 - x^4)^2\, dx$

3) $\int 3x^2 \cos(x^3)\, dx$

4) $\int -6x\, e^{2x2}\, dx$

5) $\int \frac{3}{x} \ln(x^3)\, dx$

Solutions:

1) $\frac{5(1+x^2)^4}{8} + C$

2) $\frac{(2-x^4)^3}{12} + C$

3) $\sin(x^3) + C$

4) $-\underline{\quad}3\, e^{2x2} + C$

2

5) $\frac{(\ln(x^3))^2}{2} + c$