How to find an Inverse Function

Inverse Function – A function derived from an original function in which each input becomes an output and each output because an input for the function.

Example 1: Inverse Relations:

Find the inverse of the relation:

$\{(1,2),(2,4),(7,-2),(9,-3),(10,6)\}$

Solution:

Interchange each x and y value and rewrite the relation:

$\{(\mathbf{2},\mathbf{1}),(\mathbf{4},\mathbf{2}),(\mathbf{-2},\mathbf{7}),(\mathbf{-3},\mathbf{9}),(\mathbf{6},\mathbf{10})\}$

Example 2: Inverse Functions

Find the inverse of the following function:

$f(x) = 8x + 9$

Solution:

Step 1: Replace f(x) with y:

y = 8x + 9

Step 2: Interchange x and y:

x = 8y + 9

Step 3: Solve for y (Subtract 9 from each side):

$x - \mathbf{9} = 8y + 9 - \mathbf{9}$ → $x - 9 = 8y$

Step 4: Divide by 8 on each side:

$\frac{x-9}{8} = \frac{8y}{8}$ → $\frac{x-9}{8} = y$

Step 4: Replace y with $f^{-1}(x)$:

$$\mathbf{f^{-1}(x)} = \frac{\mathbf{x-9}}{\mathbf{8}}$$

Example 3:

Find the inverse of the following function:

$$f(x) = \frac{2}{x+3}$$

Step 1: Replace $f(x)$ with y :

$$y = \frac{2}{x+3}$$

Step 2: Interchange x and y:

$x = \frac{2}{y+3}$

Step 3: Solve for y (Multiply by y + 3 on each side):

$x(y+3) = \frac{2(y+3)}{(y+3)}$ → $x(y+3) = 2$

Step 4: Divide by x on each side:

$\frac{x}{x}(y+3) = \frac{2}{x}$ → $y + 3 = \frac{2}{x}$

Step 5: Subtract 3 on each side:

$y + 3 \ \mathbf{-3} = \frac{2}{x} \mathbf{-3}$ → $y = \frac{2}{x} - 3$

Step 6: Replace y with $f^{-1}(x)$:

$\mathbf{f^{-1}(x) = \frac{2}{x} - 3}$

Practice Problems

Find the inverse of the following functions:

1) $h(x) = x^2 - 4$
2) $f(x) = \frac{4x-1}{-2x+3}$
3) $g(x) = \sqrt{5x-6}$

Solutions:

1) 1) $h^{-1}(x) = \sqrt{x+4}$
2) $f^{-1}(x) = \frac{3x+1}{2x+4}$
3) $g^{-1}(x) = \frac{y^2+6}{5}$