INTEGER EXPONENTS MULTIPLYING AND DIVIDING MONOMIALS

Important Ideas

1. Exponents can be positive numbers, negative numbers, or zero.

2. All rules that apply to positive exponents also apply to negative exponents.

3. When a monomial has been simplified all of the exponents will be positive numbers.

4. To avoid dividing by zero we make the assumption that all variables represent positive or negative numbers, but not zero.

To Divide Monomials

1. Divide the numerical coefficients.

2. Divide like bases by subtracting exponents.

We will now work through some examples where all exponents are positive numbers and the larger exponent is in the numerator.

Example 1: Simplify: $\frac{x^5}{x^2}$

Before we apply the rule for dividing like bases, we will factor out the numerator and the denominator. This will help us to see why the rule works.

$$\frac{x^5}{x^2} = \frac{x*x*x*x*x}{x*x}$$

Recall that when you are reducing fractions any factor that is in both the numerator and the denominator can be canceled or divided out. This will leave us with three factors of "*x"* in the numerator.

$$\frac{\cancel{x}*\cancel{x}*x*x*x}{\cancel{x}*\cancel{x}} = x * x * x = x^3$$

Instead of factoring we could apply the rule which says "**When dividing like bases, subtract the exponent in the denominator from the exponent in the numerator."**

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

Example 2

Let say we have the lager exponent in the denominator:

$$\frac{x^2}{x^5} = x^{2-5} = x^{-3}$$

The answer came out negative. To fix it, let us look at that situation in expending the Xs:

$\frac{\cancel{x}*\cancel{x}}{\cancel{x}*\cancel{x}*x*x*x} = \frac{1}{x*x*x} = \frac{1}{x^3}$ and it is the same as $\frac{1}{x^{5-2}}$. **Remember $\frac{x^2}{x^2}$=1, then the numerator is 1**

The variable remains where the exponent is larger, so we subtract the larger exponent from the lower. We must always come up with positive exponents in our final answers.

Example 3

$\frac{45ab^3}{15ab} = \frac{\overset{3}{\cancel{45}}\cancel{a}\cancel{b}^{\cancel{3}^{2}}}{\underset{1}{\cancel{15}}\cancel{a}\cancel{b}} = 3b^2$

Remember: Simplify the numbers (45/15= 3). Simplify the variables: "a" cancelled out and $\frac{3b^3}{b} = 3b^2$

<u>Important Rules</u>:

- Any base (except zero) that has a negative exponent can be rewritten in an equivalent form by writing a fraction where the numerator is "1" and where the denominator is the base with a positive exponent.
- Multiply like bases by adding the exponents. $\frac{36b^5b^4}{64b^3} = 2^7$ or $a^4a^3 = a^7$
- Divide like bases by subtracting the exponents. $\frac{2^7}{2^3} = 2^4$ or $\frac{a^7}{a^3} = a^4$
- <u>Any number (or expression), except zero, raised to the zero power is always equal to "1".</u> $(xy)^0 = 1$

<u>Practice.</u>

1. Simplify the following expressions:
   a. $\frac{36b^5b^4}{64b^3}$
   b. $\frac{64a^5b^4}{16ab^7}$
   c. $x^{-6} \times y^4 \times 8$
   d. $(2ad)^0$
2. Multiply the following expressions:
   a. $(4)^3\,(5)^{-2}$
   b. $(15a^7b^3)(\,4ab^6)$
   c. $(2^3)^{-4} \times (2)^7$

Answers:

1. a) $\frac{9b^6}{16}$, b) $\frac{4a^4}{b^3}$, c) $\frac{8y^4}{x^6}$, d) 1
2. a) $\frac{64}{25}$, b) $60a^8b^9$, c) $\frac{1}{32}$