FACTORING TRINOMIALS IN THE FORM OF $x^2 + bx + c$

In this standard form of a trinomial, where the coefficient of x is 1, x is a variable and b and c stand for integers.

$x^2 \quad + \quad b\,x \quad + \quad c$	
$x^2 \quad + \quad 5\,x \quad + \quad 6$	$b = 5, \quad c = 6$
$y^2 \quad - \quad 3\,y \quad + \quad 2$	$b = 3, \quad c = 2$
$a^2 \quad + \quad 3\,a \quad - \quad 40$	$b = 3, \quad c = -40$
$x^2 \quad - \quad 4\,x \quad - \quad 45$	$b = -4, \quad c = -45$

To factor this kind of polynomial means to find the two binomials which were multiplied together to get the trinomial. The method we will use is based upon **FOIL**.

$$\begin{array}{ccccccccc} & & \textbf{F} & & \textbf{O} & & \textbf{I} & & \textbf{L} \\ (x+3)(x+2) & = & (x)(x) & + & (2)(x) & + & (3)(x) & + & (3)(2) \\ & & x^2 & + & & 5\,x & & + & 6 \end{array}$$

EXAMPLE: $(x + 3)(x + 2) = (x)(x) + (2)(x) + (3)(x) + (3)(2) = x^2 + 5x + 6$

NOTE that the **L** of **FOIL** is the product of 3 and 2, while the coefficient of the middle term is the sum of 3 and 2.

We know that the **F** of **FOIL** comes from multiplying the **FIRST** two terms of the binomials.

$$x\,x\,x \cdot \ = {}^2$$

We know also that the **L** of **FOIL** comes from multiplying the **LAST** two terms of the binomials.

If the THIRD term of the trinomial is positive, then the signs of the binomials are the same. Because the **O** and **I** of **FOIL** give us a positive sum, we know that not only are the signs the same, but that they are both positive.

$$(x + \quad)(x + \quad) = x^2 + 5x + 6$$

The last thing we need to find are 2 positive factors of 6 whose sum is 5. In other words, we need to find two numbers which give us positive 6 when multiplied, but give us a positive 5 when added.

Factors of 6	Sum of the factors
1, 6	1 + 6 = 7
2, 3	2 + 3 = 5

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

EXAMPLE:

	F	O	I	L
$(y-1)(y-2) =$	$(y)(y)$ +	$[(-2)(y)]$ +	$[(-1)(y)]$ +	$[(-1)(-2)]$

$$y^2 \quad - \quad 3y \quad + \quad 2$$

NOTE that the third term of the trinomial is the product of (-1) and (-2), while the coefficient of the middle term is the sum of (-1) and (-2).
We know that the **F** of **FOIL** comes from multiplying the FIRST two terms of the binomials.

$$y\,y\cdot = y^2$$

We know that the **L** of **FOIL** comes from multiplying the LAST two terms of the binomials.
We know that because the THIRD term of the trinomial is positive, the signs of the binomials are the same. Because the **O** and **I** of **FOIL** give us a negative sum, we know that not only are the signs of the binomials the same, but that they are both negative.

$$(y - \quad)(y - \quad) = y^2 - 3y + 2$$

The last thing we need to find are two negative numbers which will give us positive 2 when multiplied, but will give us a negative 3 when added.

Negative factors of 2	Sum of the factors
$-1, -2$	$-1 + (-2) = -3$

$$(y-1)(y-2) = y^2 - 3y + 2$$

EXAMPLE:

	F	O	I	L
$(a+8)(a-5) =$	$(a)(a)$ +	$[(-5)(a)]$ +	$[(8)(a)]$ +	$[(8)(-5)]$

$$a^2 \quad + \quad 3a \quad - \quad 40$$

NOTE that the THIRD term of the trinomial is the product of 8 and -5, while the coefficient of the middle term is the sum of 8 and -5.

We know that the **F** of **FOIL** comes from multiplying the FIRST two terms of the binomials.

$$a\,a\,a\cdot = {}^2$$

We know that the **L** of **FOIL** comes from multiplying the LAST two terms of the binomials.
We know that because the THIRD term of the trinomial is negative, the signs of the binomials are different.

$$(a + \quad)(a - \quad) = a^2 + 3a - 40$$

The last thing we need to find are two numbers which will give us -40 when multiplied, but will give us +3 when added.

Factors of −40	Sum of the factors
1, −40	1 + (−40) = −39
−1, 40	−1 + 40 = 39
2, −20	2 + (−20) = −18
−2, 20	−2 + 20 = 18
4, −10	4 + (−10) = −6
−4, 10	−4 + 10 = 6
5, −8	5 + (−8) = −3
−5, 8	−5 + 8 = 3

• correct sum

Only −5 and 8 give the correct sum of 3.

$(a+8)(a$ $-5) = a^2 +$ $3a - 40$

EXAMPLE: F O I L

$(x-9)(x+$ $5) = (x)(x)$ $+ [(5)(x)]$ $+ [(-9)(x)]$ $+ [(-9)(5)]$

$$x^2 \quad - \quad 4x \quad - \quad 45$$

NOTE that the **L** of **FOIL** is the <u>product</u> of −9 and 5 and that the coefficient of the middle term is the <u>sum</u> of −9 and 5.

We know that the **L** of **FOIL** comes from multiplying the LAST two terms of the binomials.

We know that because the THIRD term of the trinomial is <u>negative</u>, the signs of the binomials will be <u>different</u>.

$$(x + \quad)(x - \quad) = x^2 - 4x - 45$$

Factors of −45	Sum of the factors
1, −45	1 + (−45) = −44
−1, 45	−1 + 45 = 44
3, −15	3 + (−15) = −12
−3, 15	−3 + 15 = 12
5, −9	5 + (−9) = −4
−5, 9	−5 + 9 = 4

• correct sum

$(x+9)(x$ $-5) = x^2 -$ $4x - 45$

<u>THINGS TO REMEMBER:</u>

1. If the constant term (third term) of the trinomial is <u>positive</u>, the signs of binomials are the same. If the middle term is positive, the signs are both positive, and if the middle term is <u>negative</u>, the signs are both negative.

$$x^2 + bx + c = (x + \quad)(x + \quad)\ x^2$$
$$-bx + c = (x - \quad)(x - \quad)$$

2. If the constant term (third term) of the trinomal is <u>negative</u>, the signs of binomials will be different.

$$x^2 + bx - c = (x + \quad)(x - \quad)\ x^2$$
$$-bx - c = (x + \quad)(x - \quad)$$

3. The coefficient of the middle term of the trinomial is the <u>sum</u> of the constant terms of the binomials.

4. The constant term of the trinomial (third term) is the <u>product</u> of the constant terms of the binomials.

A. Factor $x^2 + 11x + 18$

$x^2 + 11x + 18$

positive—the signs of the binomials will be the same.

positive—the signs will both be +.

$$(x + \quad)(x + \quad)$$

Now we need factors of 18 which add up to +11

Factors of 18	Sum of the factors	
1, 18	1 + 18 = 19	
2, 9	2 + 9 = 11	• correct sum
3, 6	3 + 6 = 9	

$$x^2 + 11x + 18 = (x + 2)(x + 9)$$

B. Factor: $y^2 - 8x + 15$

positive—the signs of the binomials will be the same

negative—the signs will both be negative.

$$(y - \ )(y - \ )$$

Now we need negative factors of 15 which add up to -8.

Factors of 15	Sum of the factors
$-1, -15$	$-1 + (-15) = -16$
$-3, -5$	$-3 + (-5) = -8$

$$y^2 - 8y + 15 = (y - 3)(y - 5)$$

Sometimes we can factor out a common factor from the trinomial before we begin to factor into two binomials.

C. <u>Factor Completely:</u>

$$4x^2 - 4x - 8$$

[The GCF is 4]

$$\frac{4x^2}{4} - \frac{4x}{4} - \frac{8}{4}$$

$$4(x^2 - x - 2)$$

Now we need to factor the trinomial into two binomials.

$$4(x^2 - x - 2)$$

coefficient is -1

Negative—the signs of the binomials will be different.

$$4(x + \ )(x - \ )$$

Now we need factors of -2 which add up to -1.

Factors of -2	Sum of the factors
$1, -2$	$1 + (-2) = -1$
$-1, 2$	$-1 + 2 = 1$

$$4x^2 - 4x - 8 = 4(x + 1)(x - 2)$$

The GCF must be in the answer.

D. <u>Factor Completely.</u>

$$3y^3 + 3y^2 - 36y$$

[GCF is $3y$] $\dfrac{3y^3}{3y} + \dfrac{3y^2}{3y} - \dfrac{36y}{3y}$

$$3\,(y\,y^2 + -y\ 12)$$

coefficient is +1

Negative, so the signs of the binomials will be different

$$3\,(y\,y + \ )(y - \ )$$

Now we need factors of −12 which add up to +1.

Factors of −12	Sum of the factors
1, −12	1 + (−12) = −11
−1, 12	−1 + 12 = 11
2, −6	2 + (−6) = −4
−2, 6	−2 + 6 = 4
3, −4	3 + (−4) = −1
−3, 4	−3 + 4 = 1

The last row of the table gives us the correct sum, so the correct factorization is

$$3\,(y\,y + 4)(y - 3)$$

EXERCISES - FACTOR COMPLETELY:

a. $x^2 - 5x + 6$

c. $y^2 + 12y + 27$

e. $y^2 + 21y - 100$

g. $ab^2 - 7ab - 8a$

i. $a^2 + 4ab - 60b^2$

g. $a\ b(+1)(b - 8)$

i. $(a + 10\ )(b\ a - 6\ )b$

b. $c^2 - c - 12$

d. $a^2 - 9z - 70$

f. $3a^2 + 15a + 18$

h. $3y^3 - 36y^2 + 81y$

j. $20x\ y^2 - 100xy + 120y$

KEY:

a. $(x - 2)(x - 3)$

c. $(y + 3)(y + 9)$

e. $(y + 25)(y - 4)$

b. $(c - 4)(c + 3)$

d. $(a - 14)(a + 5)$

f. $3(a + 2)(a + 3)$

j. $20 (y x - 2)(x - 3)$

h. $3 (y y - 3)(y - 9)$