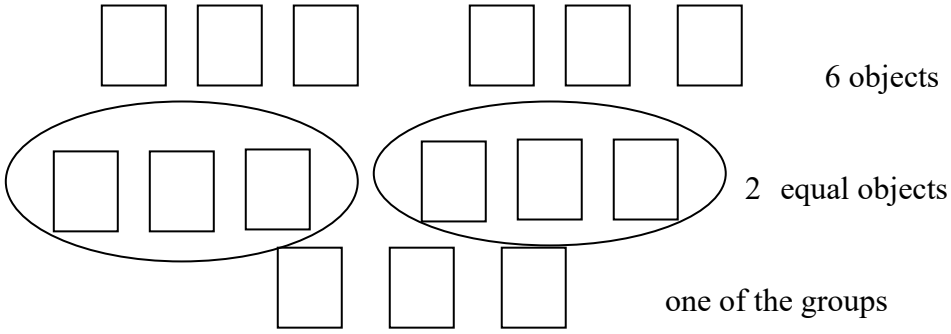
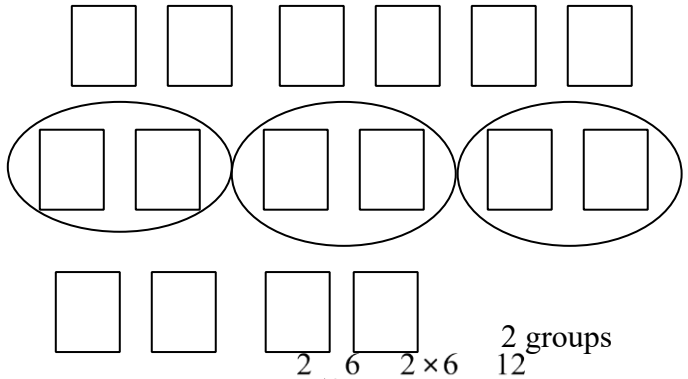
Multiplying Fractions and Mixed Numbers

You know that $\frac{1}{2}$ of 6 is 3. When you get $\frac{1}{2}$ of 6 objects, you make 2 (the denominator) equal groups of the objects and you take 1 (the numerator) of those groups.



NOTICE: $\frac{1}{2}$ of 6 is: $\frac{1}{2} \times \frac{6}{1} = \frac{1 \times 6}{2 \times 1} = \frac{6}{2} = 3$ REMEMBER: 6 is $\frac{6}{1}$.

Similary $\frac{2}{3}$ of 6 is found by separating 6 objects into 3 (the denominator) equal parts and taking two (the numerator) of those parts:



NOTICE: $\frac{2}{3}$ of 6 is 4

$\frac{2}{3} \times \frac{6}{1} = \frac{2 \times 6}{3 \times 1} = \frac{12}{3} = 4$

To Multiply Fractions: $\frac{\text{multiplynumerators}}{\text{multiplydenominators}}$; simplify

EXAMPLES:

a. $\frac{3}{8} \times \frac{7}{5} = \frac{3 \times 7}{8 \times 5} = \frac{21}{40}$

b. $\frac{2}{9} \times \frac{6}{7} = \frac{2 \times 6}{9 \times 7} = \frac{12}{63} = \frac{\cancel{3} \times 4}{\cancel{3} \times 21} = \frac{4}{21}$

EXAMPLE b. can be simplified before it is multiplied.

Write the prime factorization of each number. If the same factor appears in both the numerator and denominator, it is considered to be a factor of 1. Since 1 times any number is that same number, these common factors are eliminated and the remaining factors are multiplied.

$$\frac{2}{9}\times\frac{6}{7}=\frac{2\times(2\times3)}{(3\times\ \times3)\ 7}$$

$$=\frac{2\times2}{3\times7\times1}$$

$$=\frac{4}{21}$$

Of course, some of these steps can be done mentally! Canceling is a short cut for this method. WARNING: Many careless mistakes are made this way!

$$\frac{\cancel{2}}{\cancel{9}_{3}}\times\frac{\cancel{6}^{2}}{7}=\frac{4}{21}$$

It may be safer to write it as $\frac{\cancel{2}}{\cancel{3}\times3}\times\frac{2\cancel{3}\times}{7}=\frac{4}{21}$

1–6 MULTIPLY: (Always simplify answers.)

1. $\frac{5}{8}\times\frac{3}{10}$

2. $\times\frac{7}{12}$

3. $\frac{5}{6}\times\ \times\frac{3}{4}$

4. of 22 See the note after #6

5. $\frac{5}{9}$ of $\frac{3}{8}$

6. $\frac{1}{2}$ of $\frac{1}{2}$

See page 1 of this lab. A fraction "OF" a number is the fraction "TIMES" the number.

In an earlier section you learned to write a mixed number as a fraction. When mixed numbers are factors, you first write them as a fraction.

EXAMPLES: a. $\frac{1}{3}$ of $5\frac{1}{4}$

$$\frac{1}{3} \times \frac{21}{4}$$

$$\frac{1 \times 7 \times \cancel{3}^{1}}{\cancel{3}_{1} \times 4} = \frac{7}{4} \quad 1\frac{3}{4}$$

(NOTICE 4 wasn't written as 2 × 2 since there were no factors of 2 in the numerator.)

b. $6 \times 7\frac{1}{2}$

$$\frac{20}{3} \times \frac{15}{2}$$

$$\frac{(\cancel{2} \times 2 \times 5) \times (3 \times \cancel{5})}{3 \times \cancel{2}_{1}} = \frac{50}{1} = 50$$

WARNING: Do not treat multiplication of mixed numbers like addition of mixed numbers.

REMEMBER $6\frac{2}{3}$ means $6 + \frac{2}{3}$ and $7\frac{1}{2}$ means $7 + \frac{1}{2}$.

$6\frac{2}{3} \times 7\frac{1}{2}$ is $6 + \frac{2}{3} \times 7 + \frac{1}{2}$

The Commutative and Associative Properties, which allow us to change the order and regroup, do not apply here because there is addition and multiplication in this problem.

We know the answer of $\frac{2}{3}$ $\frac{1}{2}$ 6×7 is 50. If we found $6 \cdot 7$ and $\frac{2}{3} \times \frac{1}{2}$, we would only have 42,

which is NOT THE ANSWER.

We can use this method to add because the problem contains all addition.

$$1\frac{2}{3} = 13 + \frac{4}{6} + \frac{3}{7} = 13 + \frac{28}{42} + \frac{18}{42} = 13 + \frac{46}{42} = 13 + 1\frac{4}{42} = 14\frac{4}{42}$$

The sum is $14\frac{1}{6}$. (Read again why addition CAN be done this way but multiplication cannot.)

7–15. Find the products:

7. $2\frac{4}{5} \times 5\frac{1}{7}$

8. $\frac{2}{3}$ of 79. $12 \times 2\frac{1}{2}$

$\frac{3}{8}$

10. $\frac{1}{4} \times 1\frac{1}{5} \times 3\frac{1}{10}$

11. $6\frac{2}{3} \times 4\frac{1}{8}$

12. $83\frac{1}{3} \times 1\frac{1}{100}$

13. $\frac{1}{2} \times 100 \times \frac{11}{100} \times \frac{1}{2}$

14. $\frac{1}{2} \times \frac{1}{100}$

15. $62\frac{1}{2} \times \frac{1}{100}$

16–20. Answer the following questions.

16. What is the rule for multiplying fractions?

17. If there is a whole number in a multiplication problem which also has fractions, explain whatmust be done.

18a. If there is a mixed number as a factor in a multiplication problem, what must be done before you multiply?

18b. Explain why you can't just group whole numbers and group fractions in a multiplication problem as we did when we added.

19. Do you need to have common denominators to multiply fractions?

20. Explain why you can "cancel" common factors in the numerator and denominator of fractionsthat are multiplied but not of fractions that are added or subtracted.

ANSWERS:

1. $\frac{3}{16}$ 2. $3\frac{1}{2}$ 3. $\frac{3}{20}$ 4. $16\frac{1}{2}$ 5. $\frac{5}{24}$ 6. $\frac{1}{4}$ 7. $14\frac{2}{5}$

8. 5 9. $28\frac{1}{2}$ 10. $\frac{3}{200}$ 11. $27\frac{1}{2}$ 12. $\frac{5}{6}$ 13. 50 14. $\frac{1}{200}$

$\frac{5}{8}$ 15. 16. Mul tiply numerators 17. Write the whole number over 1 Multiply denominators making it a fraction with the same Simplify (You **can** simplify first): value!

EXAMPLE: $3 = \frac{3}{1}$

18. a. Mixed numbers are written as fractions. Multiply whole number × denominator then add numerator. This is the numerator of the improper fraction. Keep the original denominator.

b. The Associative Law for Multiplication works when **all** operations are multiplication. A mixed number is a short way of writing **addition** of a whole number and a fraction.

$6\frac{2}{5} \times 31$ is not just multiplication of the numbers we see. It is $6 + \frac{2}{5} \times 31 + \frac{1}{4}$.

19. No. Common denominators are not needed to multiply.

20. A common factor in the numerator and denominator is a factor of 1. The Multiplication Property of One says multiplying 1 × a number does not change the number.

EXAMPLE: $\frac{2}{15} \times \frac{10}{3} = \frac{2\times(5\times 2)}{(3\times 5)\times 3}$. This is the same as $\frac{2}{3} \times \frac{2}{3} \times 1$ or $\frac{4}{9}$.

Also notice when we have common denominators we still multiply them. We always use the rule for multiplication of fractions to multiply fractions no matter what their denominators are!

In $\frac{2}{15} + \frac{10}{3}$, the common factors of 5 are not in factors, but are in addends of the problem, so we cannot use the property used in multiplication.