Simplifying Algebraic Fractions

In arithmetic, you learned that a fraction is in simplest form if the Greatest Common Factor (GCF) of the numerator and the denominator is 1. Sometimes it was helpful to write the prime factorization of the numerator and denominator to find the GCF.

Example: Simplify $\frac{18}{24}$

$$\frac{18}{24} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} = \frac{3}{4}$$

Since $\frac{2}{2} = 1$ and $\frac{3}{3} = 1$, using the Multiplication Property of One, we say $1 \cdot 1 \cdot \frac{3}{2 \cdot 2} = \frac{3}{4}$.

Instead, we could have reduced using the GCF = 6:

$$\frac{18}{24} = \frac{6 \cdot 3}{6 \cdot 4} = \frac{3}{4}$$

It is very important to know that we were working with <u>factors</u>. The numerator was <u>all multiplication</u> and the denominator was <u>all multiplication</u>. **To simplify fractions, we must <u>factor first</u>!**

Now let's look at algebraic fractions. If a fraction has a polynomial in the numerator and a polynomial in the denominator, it is an algebraic fraction.

Examples of algebraic fractions:

1. $\frac{3x+5}{x-8}$ 2. $\frac{4}{c}$ 3. $\frac{x^2-6x+9}{2x-6}$

Restricted Values

When we work with algebraic fractions, we must be sure that the denominator is not equal to zero. Remember, we do not divide by zero. $\frac{5}{0}$ is undefined. It is not a real number. There are restrictions that must be placed on the variables so that the denominators will not be zero. Look at example 2 above. It is easy to see that if $c = 0$, the fraction would not be a real number. We say that $c \neq 0$ in example 2.

It is not quite that easy to see the restrictions in examples 1 and 3. Think about example 1—if the denominator is 0, $x - 8 = 0$. We can solve this equation to see the value of x that would cause $x - 8$ to be zero.

$$x - 8 = 0$$
$$x - 8 + 8 = 0 + 8$$
$$x = 8$$

Since we do not want $x - 8$ to be 0, we say $x \neq 8$.

Look at example 3. If $2x - 6 = 0$, this expression would not represent a real number. Find the restriction for x.

Solve the equation $2x - 6 = 0$. The solution is $x = 3$. This tells us if $x = 3$, the denominator is zero. We do not want the denominator to be zero, so write, $x \neq 3$.

To simplify algebraic fractions, we must <u>factor first</u>!

Example: Simplify $\dfrac{x^2 - 6x + 9}{2x - 6}$. **Don't even think about simplifying until you have factored!**

$$\begin{array}{rc} \text{Factor the numerator:} & (x-3)(x-3) \\ \hline \text{Factor the denominator:} & 2(x-3) \end{array}$$

The numerator is **(binomial)** · **(binomial)**. It is all multiplication. Remember the parentheses are symbols of grouping. We treat each binomial as one factor—not as two terms! The denominator is **(monomial)** · **(binomial)**. It is all multiplication.

When we have two identical factors, we can use the Multiplication Property of One to simplify just as we did in arithmetic.

Since $\dfrac{(x-3)}{(x-3)} = 1$, we can write $\dfrac{(x-3)}{2} \cdot \dfrac{(x-3)}{(x-3)} = \dfrac{(x-3)}{2} \cdot 1 = \dfrac{1(x-3)}{2}$ or just $\dfrac{x-3}{2}$

Restricted Values (again)

Earlier, we saw why x cannot be 3 by solving the equation $2x - 6 = 0$. You can easily find the restrictions of the variables after you factor the denominator. If either factor in the denominator is zero, because of the Multiplication Property of Zero, the denominator would be zero. When we factored the denominator $2x - 6$, the factors were 2 and $(x - 3)$. The first factor does not have a variable part and it is not zero itself. Let's let the second factor be zero and solve for x.

$$x - 3 = 0$$
$$x - 3 + 3 = 0 + 3$$
$$x = 3$$

Remember this tells us that if $x = 3$, the denominator will be zero. Since we do not want the denominator to be zero, we write $x \neq 3$.

From this point on, we will *assume* that the variables are not equal to their restricted values so that we can concentrate only on simplifying the algebraic fractions.

Examples: Simplify each of the following.

1. $\dfrac{8x^3y^7}{12xy^9}$ 2. $\dfrac{3-x}{x-3}$ 3. $\dfrac{25-x^2}{x^2-2x-15}$

Remember, to simplify algebraic fractions, we must <u>factor first</u>!

1. This example is a monomial divided by a monomial. We can use the rules for simplifyingfractions and the exponential rules for dividing.

$$\frac{8x^3y^7}{12xy^9} = \frac{2\cdot2\cdot2}{2\cdot2\cdot3}\cdot x^{3-1}\cdot y^{7-9} = \frac{2}{3}x^2y^{-2} = \frac{2x^2}{3y^2}$$

2. There is a binomial in the numerator and a binomial in the denominator. Their GCF = 1. Wedo see that these binomials are identical except for the signs. We can factor $3 - x$ as $-1(x - 3)$. If you use the Distributive property, you will get $-x + 3$, which is equivalent to $3 + (-x)$ or simply $3 - x$, the original numerator. When we factor out the -1, our binomial factors are identical. The denominator is the same as $1(x - 3)$.

$$\frac{3-x}{x-3} = \frac{-1\cancel{(x-3)}}{1\cancel{(x-3)}} = \frac{-1}{1} = -1$$

A number divided by its opposite is -1. A polynomial divided by its opposite is -1.

3. $\dfrac{25-x^2}{x^2-2x-15}$ **Factor first!** Factor the numerator and factor the denominator.

$$\frac{25-x^2}{x^2-2x-15} = \frac{(5-x)(5+x)}{(x-5)(x+3)}$$

The factors in the numerator are not the same as the factors in the denominator, but $(5 - x)$ is the opposite of $(x - 5)$, so we will factor out -1.

$$\frac{-1(x-5)(x+5)}{(x-5)(x+3)}$$ Notice, we have used the Commutative Property of Addition to write $(5 + x)$ as $(x + 5)$ and $(-5 + x)$ as $(x - 5)$.

$$= \frac{-1(x+5)}{(x+3)}$$ Since $\frac{(x-5)}{(x-5)} = 1$

This answer can be written as $\frac{-(x+5)}{(x+3)}$ or $-\frac{x+5}{x+3}$. The negative can be written in front of the fraction bar. The parentheses are not needed because the fraction bar is also a symbol of grouping. If we use the Distributive Property to clear the parentheses in the answer $\frac{-(x+5)}{(x+3)}$, we will have $\frac{-x-5}{x+3}$, which is also an acceptable answer.

Let's study one more example. Then try the exercises that follow.

Example: $$\frac{12x^3 + 42x^2 - 24x}{4x^3 - 64x^2}$$ To simplify this algebraic fraction, we must look for common factors. We must **factor first!**

$$\frac{6x(2x^2+7x-4)}{4x(x^2-16)}$$ Find GCF first.

$$\frac{6x(2x-1)(x+4)}{4x(x+4)(x-4)}$$ There are several common factors in the numerator and denominator. They are 2, x and $(x + 4)$. Simplify.

$$\frac{3(2x-1)}{2(x-4)}$$ We may leave the answer in factored form.

Exercises:

SIMPLIFY. Remember to factor first! Notice that problem 1 and 2 are already in factored form. The numerators are all multiplication. The denominators are all multiplication. Because of the parentheses, each binomial is treated as one factor, not as two terms.

_______ 1. $-12x^3\,{}^2y$ 2.3. x^2 $\dfrac{(3x-2)(x+4)}{(3x-2)(x+6)}$ $+11x + 28$ __________

$16xy^5x^2 + 3x - 28$

4. $6x^2 + x - 15$

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$9x^2 - 25$

5. 3 $4x$ −

_____

8 $6 - x$

6. $2x^2 + 16x + 2$[1]

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$6x^2 + 36x + 48$

7. $10x^2 + 9x - 9$

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[1] $4y^3$ $x + 6$ $x - 4$ 3 $5x$ − 2 $3(x+4)$ 2 $1x$ +

.

$3 + x - 10x^2$

Answers

1. $-3x^2$ 2. $x + 4$ 3. $x +$ 4 4. 2 $3x -$ 5.

$-$ 1 6. $x+6$ 7. $- 2$ $3x +$