Simplifying Rational Expressions

A *rational expression* is a quotient of two monomials and/or polynomials.

Examples: $\frac{3x+1}{2}$, $\frac{x^2-1}{5x^2+2x}$, $\frac{3y}{5y^2}$, $\frac{1}{2}$, etc.

<u>Beginning Steps</u>:

1. Factor out the greatest common factor (**GCF**), if any, in all numerators and denominators.
2. **Factor completely** all numerators and denominators.
3. Remaining steps depend on the type of problem. See examples below for three types of problems: (I) single expression; (II) products and quotients; (III) sums and differences.

<u>Type I: Single Rational Expression</u>

Example #1 - Simplify: $\frac{w^2+w-12}{w^2+8w+16}$.

Solution -

$\frac{w^2+w-12}{w^2+8w+16}$ 1. there is no GCF in either numerator or denominator

$= \frac{(w-3)\,\mathbf{(w+4)}}{(w+4)\,\mathbf{(w+4)}}$ 2. factor numerator and denominator completely

$= \boxed{\frac{(w-3)}{(w+4)}}$ 3. cancel any like factors (representing 1)

Example #2 - Simplify: $\frac{3x+6}{3x}$.

Solution -

$\frac{3x+6}{3x}$

$= \frac{3(x+2)}{3x}$ 1. factor out GCF (of 3) in the numerator

$= \frac{\mathbf{3}}{\mathbf{3}} \cdot \frac{(x+2)}{x}$ 2. there's no other factorization to be done

$= \boxed{\frac{x+2}{x}}$ 3. cancel like factors (representing 1)

<u>Note</u> - it is **incorrect** to cancel as follows: $\frac{\mathbf{3x}+6}{\mathbf{3x}} \neq 7$ since the 3x in 3x+6 is not a factor but a term of the sum: 3x+6.

However, $\frac{\mathbf{3}(x+2)}{\mathbf{3}x} = \frac{x+2}{x}$ is correct.

Example #3 - Simplify: $\frac{150x-6x^3}{6x^2-27x-15}$.

Solution -

$\frac{150x-6x^3}{6x^2-27x-15}$

$= \frac{6x(25-x^2)}{3(2x^2-9x-5)}$ 1. factor out GCF of 6x in numerator and of 3 denominator

$= \frac{3 \cdot 2x(5+x)(5-x)}{\ldots}$ 2. factor numerator and denominator completely

3(2x + 1) (x - 5)

3• 2x (5 + x) (-1) **(x - 5)**

=3. cancel like factors (representing 1)

3(2x + 1) **(x - 5)**

= $\frac{-2x(5 + x)}{2x + 1}$

Note : 5 - x = **-1**(-5 + x) = **-1**(x - 5). (factoring out -1 and rewriting)

Type II: Products and Quotients

Example #4 - Simplify: 4x + 1x - 9 ÷ 10xx2 + 8x + 7 + 19x + 6 • 52x + x - 103 .

$4x^2$ - 9 $10x^2$ + 19x + 6 5x + 10

Solution - x + 1 ÷ x^2 + 8x + 7 • 2x - 3

= $\frac{4x^2 - 9}{x + 1}$ • $\frac{x^2 + 8x + 7}{10x^2 + 19x + 6}$ • $\frac{5x + 10}{2x - 3}$ (i) change division to mult. by the reciprocal

= ~~(2x + 3) (2x - 3)~~ / **x + 1** • (x + 7) **(x + 1)** / **(2x + 3)** (5x + 2) • 5(x + 2) / **2x - 3** (ii) factor completely and cancel

=(iii) $\frac{5(x + 2)(x + 7)}{5x + 2}$ write final answer as one fraction

Example #5 - Simplify: $\frac{3x - 1}{2x^2 - 9x - 5}$ • $\frac{x^2 + 2x - 8}{6x - 2}$.

Solution - 2x23 - 9x - 5x - 1 • x2 + 2x - 86x - 2

= ~~3x - 1~~ / (2x + 1) (x - 5) • ~~(x + 4) (x - 2)~~ / 2**(3x - 1)** (i) factor completely

=(ii) $\frac{(x + 4)(x - 2)}{2(2x + 1)(x - 5)}$ cancel like factors

Type III: Sums and Differences

Example #6 - Add and simplify: y^2 + 12y + 203y + y^2y + 7 - 100 (i) factor denominators & find LCD

Solution - 3y / (y+10) (y+2) + y + 7 / (y + 10) (y - 10)

(ii) LCD is (y +10)(y + 2)(y - 10)

= ~~3y~~ / (y+10) (y+2) • **(y - 10)** / **(y - 10)** + y + 7 / (y + 10) (y - 10) • ~~(y + 2)~~ / **(y + 2)** (iii) write all fractions with LCD

= ~~$3y^2$ - 30y~~ / ~~(y + 10) (y + 2) (y - 10)~~ + ~~y^2 + 9y + 14~~ / (y + 10) (y + 2) (y - 10) (iv) add fractions

=(v) $\frac{3y^2 - 30y + y^2 + 9y + 14}{(y + 10)(y + 2)(y - 10)}$ if possible, factor and reduce

$$= \boxed{\frac{4y^2 - 21y + 14}{(y + 10)(y + 2)(y - 10)}}$$ (vi) (cannot factor or reduce further)

Example #7 - Subtract and simplify: $\frac{x + 3}{2x^2 + 13x + 6} - \frac{5}{6x + 3}$

Solution - $\frac{x + 3}{2x^2 + 13x + 6} - \frac{5}{6x + 3}$ (i) factor all denominators and find LCD

$$= \frac{x + 3}{(2x + 1)(x + 6)} - \frac{5}{3(2x + 1)}$$ (ii) LCD is: $3(2x + 1)(x + 6)$

$$= \frac{x + 3}{(2x + 1)(x + 6)} \cdot \left(\frac{\mathbf{3}}{\mathbf{3}}\right) - \frac{5}{3(2x + 1)} \cdot \frac{\mathbf{(x + 6)}}{\mathbf{(x + 6)}}$$ (iii) write all fractions with LCD

$$= \frac{3x + 9}{3(2x + 1)(x + 6)} - \frac{5x + 30}{3(2x + 1)(x + 6)}$$ (iv) subtract fractions

$$= \frac{3x + 9 - 5x - 30}{3(2x + 1)(x + 6)} = \boxed{\frac{-2x - 21}{3(2x + 1)(x + 6)}}$$ (v) (cannot factor or reduce further)