

Graphing Linear Equations

A linear equation has infinitely many ordered pair solutions. The graph of an equation in two variables is a drawing of the ordered pair solutions of the equation. It is not possible to name *all* the solutions. We generally find three ordered pair solutions and graph them. The complete solution set can be shown by drawing a straight line through the graphs of the ordered pairs. An arrow on each end of the line shows that the solution set continues in both directions.

EXAMPLE: Graph $y = 3x + 2$

To find three ordered pair solutions, pick **any** three values for x and solve for y .

Let $x = 1$:

$$y = 3(1) + 2$$

$$y = 3 + 2$$

$$y = 5$$

$$(1, 5)$$

Let $x = -1$:

$$y = 3(-1) + 2$$

$$y = -3 + 2$$

$$y = -1$$

$$(-1, -1)$$

Let $x = -2$:

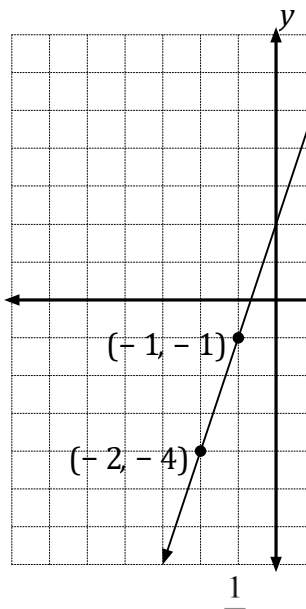
$$y = 3(-2) + 2$$

$$y = -6 + 2$$

$$y = -4$$

$$(-2, -4)$$

Now we graph the ordered pair solutions $(1, 5)$, $(-1, -1)$, and $(-2, -4)$.



NOTE that the three points fall in a straight line. **EVERY** point on the line is a solution of the equation and can be represented by an ordered pair. Two points are sufficient to draw a straight line, but we generally get a third point as a check.

EXAMPLE: Graph $y = -\frac{1}{4}x + 1$

NOTE that the coefficient of x in this equation is a fraction. When this occurs we want to pick values for x which will allow us to eliminate the fraction. As the denominator of the fraction is 4, the easiest choices to work with will be multiples of 4, such as 0, 4 and -4.

Let $x = 0$:

$$y = -\frac{1}{4}(0) + 1$$

$$y = 0 + 1$$

$$y = 1$$

$$(0, 1)$$

Let $x = 4$:

$$y = -\frac{1}{4}(4) + 1$$

$$y = -1 + 1$$

$$y = 0$$

$$(4, 0)$$

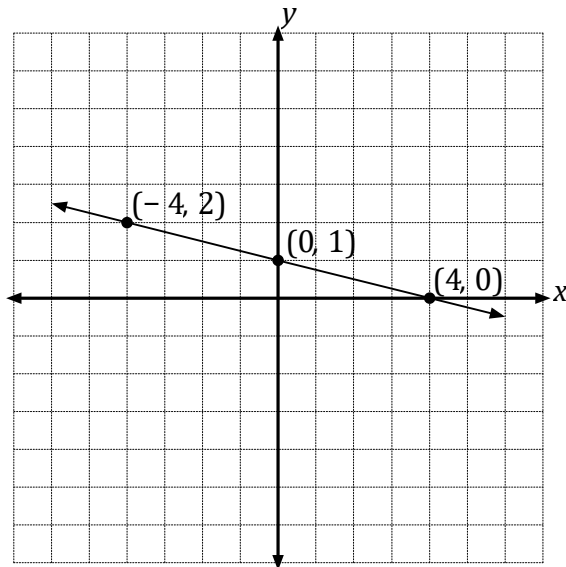
Let $x = -4$:

$$y = -\frac{1}{4}(-4) + 1$$

$$y = 1 + 1$$

$$y = 2$$

$$(-4, 2)$$



Any time the coefficient of x is a fraction, convenient choices for x are zero, the denominator, and the opposite of the denominator.

$$y = -\frac{2}{3}x + 8 \quad \text{choose } 0, 3, \text{ and } -3$$

$$y = -\frac{2}{5}x + 4 \quad \text{choose } 0, 5, \text{ and } -5$$

Sometimes the equation is in the form of $Ax + By = C$, and in this case we can solve the equation for y first.

EXAMPLE: Graph $2x + 3y = 12$

To solve the equation for y , follow these steps:

$$2x + 3y = 12$$

1. To isolate the y term, add the opposite of the term containing x to both sides of the equation.

$$2x - 2x + 3y = -2x + 12$$

$$3y = -2x + 12$$

2. Divide both sides of the equation by the coefficient of y . $3y = -2x + 12$ This means

both terms on the right-hand side must be $3 \quad 3 \quad 3$

divided by the coefficient. $2y = -\frac{2}{3}x + 4$

Once the equation is in the form of $y = mx + b$, the ordered pair solutions can be found by picking values for x and solving for y . As the coefficient of x is $-\frac{2}{3}$ we would pick 0, 3 and -3 to get the ordered pairs.

Let $x = 0$:

$$y = -\frac{2}{3}(0) + 4$$

$$y = +0 + 4$$

$$y = 4$$

$$(0, 4)$$

Let $x = 3$:

$$y = -\frac{2}{3}(3) + 4$$

$$y = -2 + 4$$

$$y = 2$$

$$(3, 2)$$

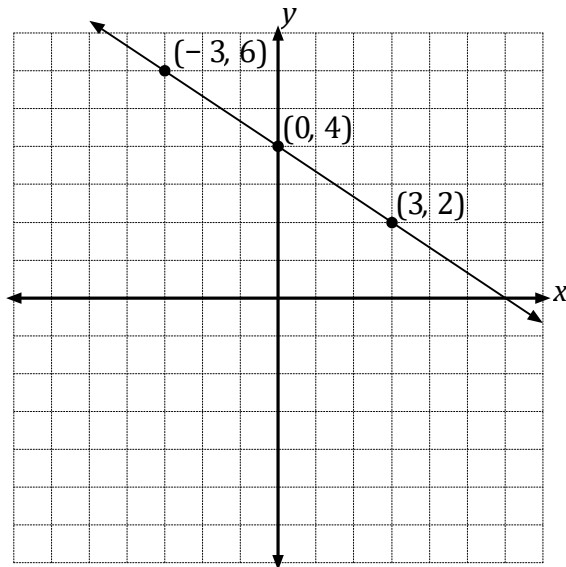
Let $x = -3$:

$$y = -\frac{2}{3}(-3) + 4$$

$$y = +2 + 4$$

$$y = 6$$

$$(-3, 6)$$



Now we will practice rewriting equations in the form $Ax + By = C$ to their equivalent $y = mx + b$ form.

EXAMPLE: Solve for y : $x + 4y = -6$

$$\begin{aligned}
 x + - + = - - (x) \\
 4y \quad x \\
 6 \\
 4y = - - -x6 \\
 4 \quad x \\
 6 - y \\
 = - - - \\
 4 \quad 4 \quad 4 \\
 1 \quad 3 \\
 y = - x - \frac{1}{4} - \frac{3}{4} \\
 4 \quad 2
 \end{aligned}$$

Add $-x$ to both sides

Divide each term by 4

EXAMPLE: Solve for y : $-2x - 4y = 8$

$$\begin{aligned}
 - + - = +2x2x 4y \\
 2x8 \\
 - = +4y 2x \\
 8
 \end{aligned}$$

Add $2x$ to both sides

$$\begin{aligned}
 -4 2x 8 \frac{y}{-4} = + - \\
 -4 \frac{-4}{-4} -4 \frac{y}{-4} = - \frac{8}{-4}
 \end{aligned}$$

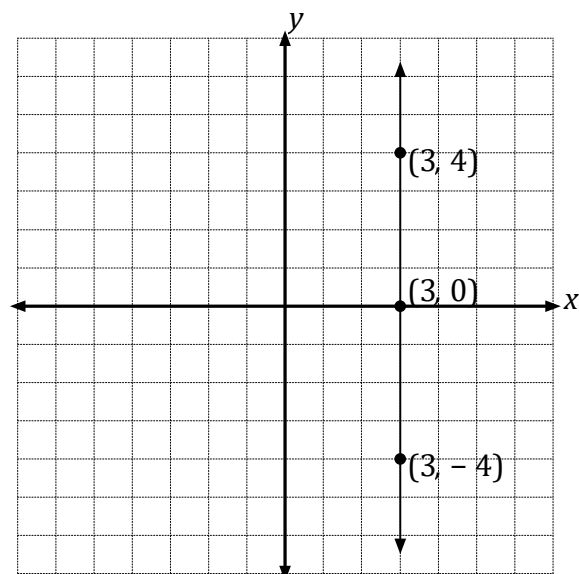
Divide each term by -4

$$\begin{aligned}
 & \frac{1}{2} \\
 x - 2
 \end{aligned}$$

EXAMPLE: Graph the equation $x = 3$.

NOTICE that the equation $x = 3$ does not mention y . This equation could be written as $0 \cdot y + x = 3$. In this case no matter what value y has, because y is multiplied by 0, x will always be 3. This graph will be a **vertical line** through the point where $x = 3$.

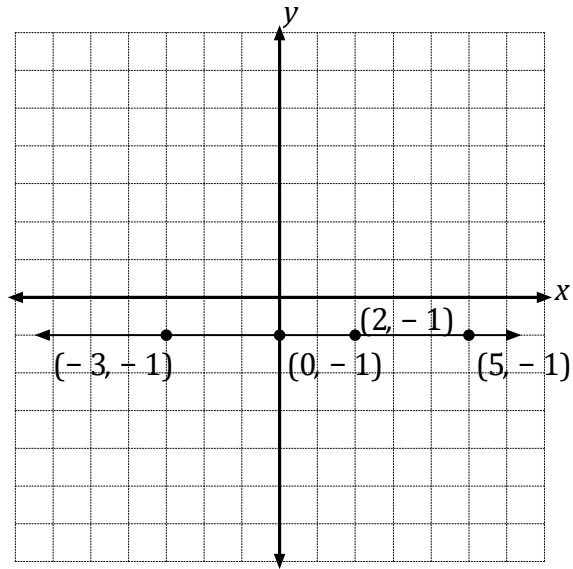
x	y
3	-4
3	-2
3	0
3	2
3	4



EXAMPLE: Graph the equation $y = -1$.

NOTICE that this equation does not mention x . This equation could be written as $0 \cdot x + y = -1$. In this case no matter what value x has, because x is multiplied by 0, y will always be -1 . This graph will be a **horizontal line** through the point where $y = -1$.

x	y
-3	-1
-1	-1
0	-1
2	-1
5	-1



PRACTICE

Graph the lines of the following equations. Find at least three ordered pairs associated with each line.

a. $3x - y = 4$

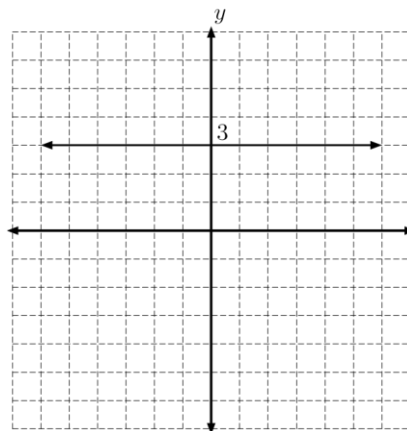
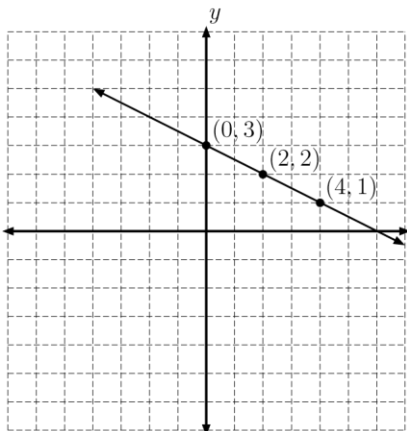
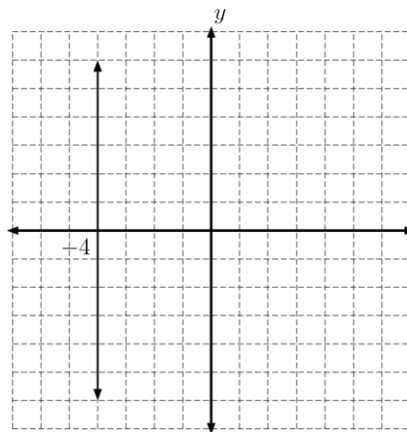
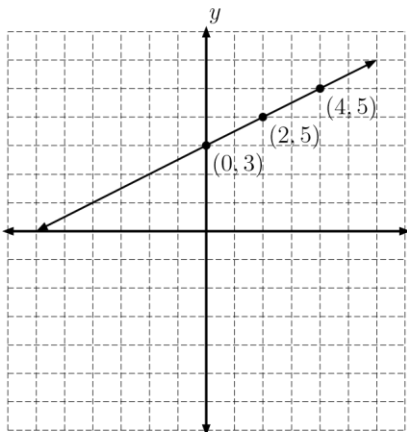
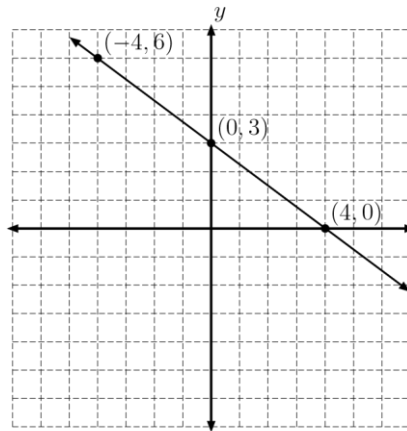
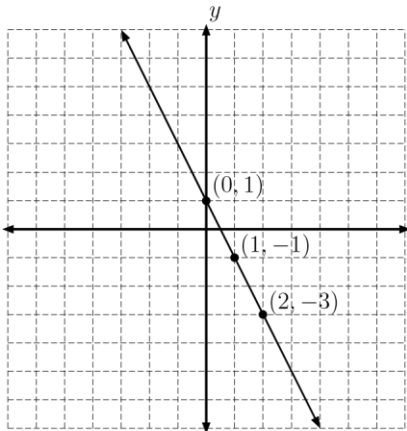
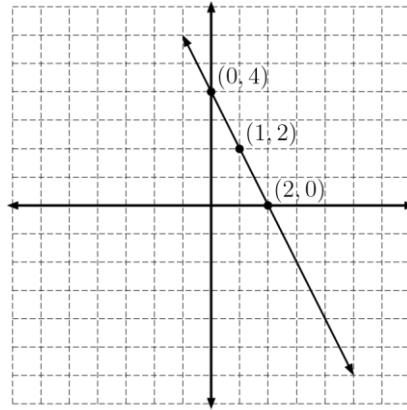
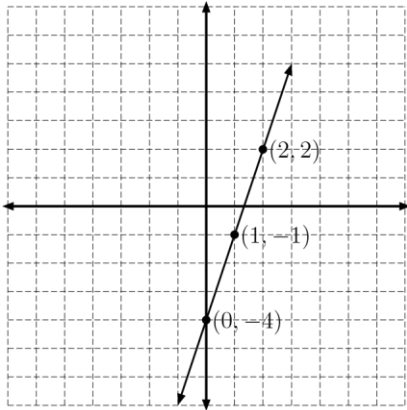
e. $2x + y = 4$

b. $2x + y = 1$

f. $3x + 4y = 12$

c. $y = \frac{1}{2}x + 3$ g. $x = -4$

d. $y = -\frac{1}{2}x + 3$ h. $y = 3$



a. y e. y

b.f.

c.g.

d.h.