Solving Linear Inequalities and Compound Inequalities

Steps for solving linear inequalities are very similar to the steps for solving linear equations. The big differences are multiplying and dividing a constant on the inequalities and expressing the solution set. **However, if you want to practice with solving linear equations, you can refer to the previous topic. (Topic 6)** This handout will show some examples on how to solve linear inequalities and compound inequalities and how to express the solution sets of inequalities.

Solve Linear Inequalities

Example (1): $3\ 8\ 6x+\ >$

Solution:

$3\ 8\ 8\ 6\ 8x+\ -\ >\ -$ — Subtract 8 on each side

$\frac{3}{3}x > -2\ \frac{3}{3}x>$ — Divide 3 on each side. **Do not reverse** the inequality symbol.

$\frac{-2}{3}\ x> \frac{-2}{3}$ — Place the solution set in the set-builder notation

The solution set is $\left\{x \mid x > -\frac{2}{3}\right\}$

Example (2): $3\ 2\ 5\ 13x-\ \geq\ x+$

Solution:

$3\ 2\ 2\ 5\ 13\ 2x-\ +\ \geq\ x+\ +$ — Add 2 on each side

$3x\geq\ +5x\ 15$ — Simplify

$3\ 5\ 5\ 5\ 15x-\ x\geq\ x-$ — Subtract 5x on each side

$+x\ -2\ 15x\geq$ — Simplify

$\frac{-2x}{-2} \leq \frac{15}{-2}$ — Divide -2 on each side; **reverse** the inequality symbol (when divide or multiply a negative number)

$x \leq -\frac{15}{2}$

The solution set is $\left\{x \mid x \leq -\frac{15}{2}\right\}$ — Place the solution set in the set-builder notation.

Example (3): $6(3+4x)-2<20$

Solution:

$18+24x-2<20$

$24x+16<20$

$24x+16-16<20-16$

$24x<4$

$\frac{24x}{24}<\frac{4}{24}$

$x<\frac{1}{6}$

The solution set is $\left\{x \mid x<\frac{1}{6}\right\}$

Example (4): $\frac{1}{2}(w-3)-(2-w)\leq 1$

Solution:

$(2)\frac{1}{2}(w-3)-(2)(2-w)\leq(2)1$

$(w-3)-2(2-w)\leq 2$

$w-3-4+2w\leq 2$

$3w-7\leq 2$

$3w-7+7\leq 2+7$

$3w\leq 9$

$w\leq 3$

The solution set is $\{w \mid w\leq 3\}$

Remove the parenthesis by multiplying 6 to 3 and 4x.

Simplify

Subtract 16 on each side

Simplify

Divide 24 on each side. **Do not reverse** the inequality symbol.

Simplify

Place the solution set in the set-builder notation

Multiply 2 on the inequality **each term** to simplify

Simplify

Remove parenthesis. Multiply -2 to

$(2 - w)$

Simplify

Add 7 on each side

Simplify

Divide 3 on each side. **Do not reverse** the inequality symbol.

$$z < -\frac{11}{5}$$

The solution set is $\left\{ z \mid z < -\frac{11}{5} \right\}$

Place the solution set in the set-builder notation

Example (5): $\frac{5z-4}{5} > \frac{2+5z}{3}$

Interval Notation

Find LCD=15. Multiply 15 to each term

Solution: $(15)\frac{5z-4}{5} > (15)\frac{2+5z}{3}$

Simplify

$$3(5z-4) > 5(2+5z)$$

Distribute property to remove the parenthesis Add 12 on each side

$$15z-12 > 10+25z$$

Simplify

$$15z-12+12 > 10+12+25z$$

Subtract 25z on each side

$$15z > 22+25z$$

Simplify

$$15z-25z > 22+25z-25z$$

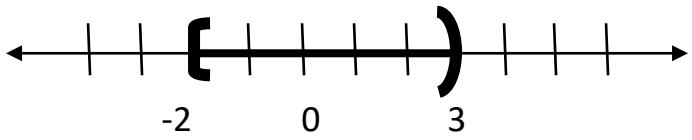
$$-10z > 22$$

Divide -10 on each side. **Reverse** the inequality symbol.

$$-\frac{10}{10}z < -\frac{22}{10}$$

Simplify

Place the solution set in the set-builder notation



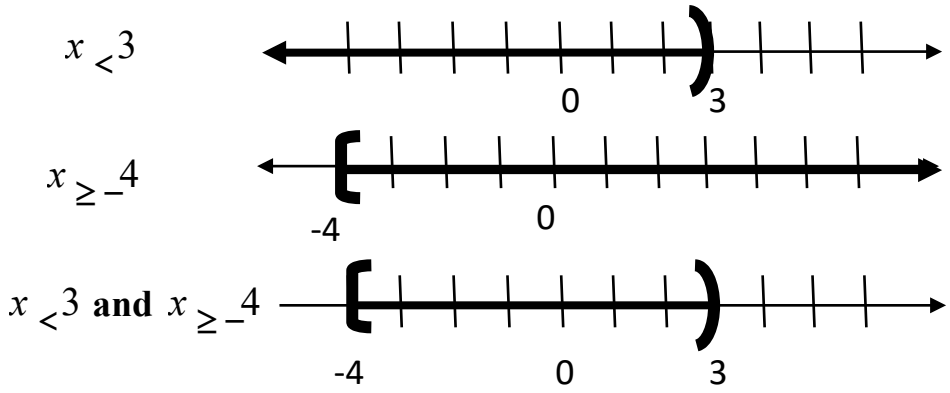
$[-2,3)$

Use the open parentheses ( ) if the value is not included in the graph, i.e. greater than (>) or less than (<). Use the brackets [ ] if the value is part of the graph, i.e. greater than or equal to (≥). Whenever there is a break in the graph, write the interval up to the point. Then write another interval for the section of the graph after that part. Put a union sign "∪" between each interval to "join" them together.

Solve Compound Inequalities (two inequalities joined by "and" or "or")

Example (1): $x<3$ **and** $x\geq -4$

Solution: When solving compound inequalities, we usually graph them on the number lines to get the solution set.



* When two inequalities joined by "and", that means **interception** of the solutions.

* Graph the inequalities separately.

* Look for overlapping of the graph.

* What you see is what you get.

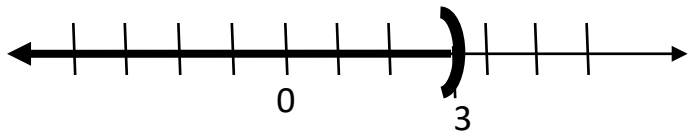
Write out the interval notation from the overlapping segment, if any.

Interval Notation: $[-4,3)$

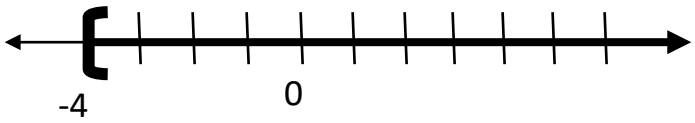
Example (2): $x<3$ **or** $x\geq -4$

Solution: When solving compound inequalities, we usually graph them on the number lines to get the solution set.

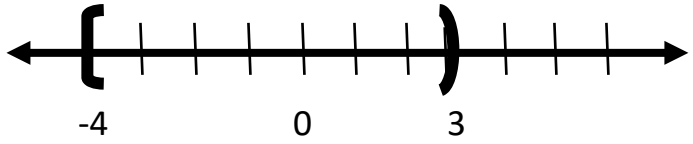
$x < 3$



$x \geq -4$



$x < 3$ **or** $x \geq -4$



When two inequalities joined by "or", that means **union** of the solutions.

* Graph the inequalities separately.

* Look for everything shaded on the graph.

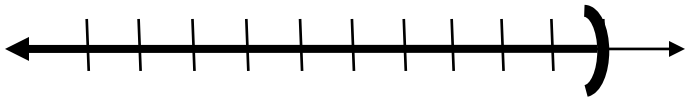
* What you see is what you get. Write out the interval notation from the number line.

Interval Notation: $(-\infty, \infty)$

Example (3): $x+1<9$ **and** $2x-1>7$

Solution: We need to solve each inequality before we can place them on the number lines.

$x+1-1<9-1$



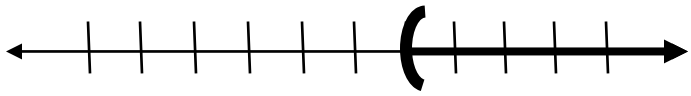
$x<8$

When two inequalities joined by "and", that means **interception** of the solutions.

$2x-1+1>7+1$ *

$2x>8$

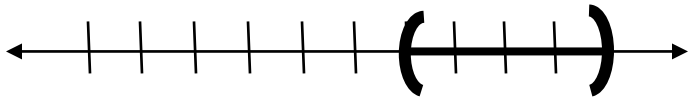
$x>4$



Graph the inequalities separately.

* Look for overlapping of the graph.

* What you see is what you get.



$x<8$ **and** $x>4$

Write out the interval notation from the overlapping segment, if any.

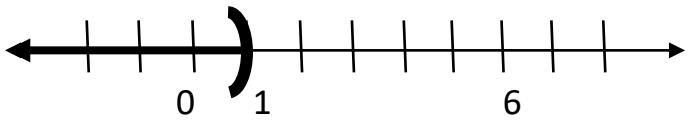
Interval Notation: $(4,8)$

Example (4): $x+1<2$ **or** $2x-1>8$

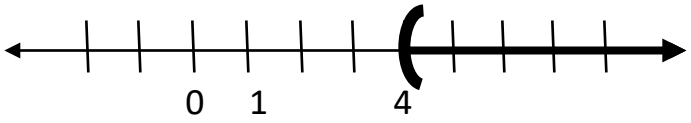
Solution: We need to solve each inequality before we can place them on the number lines.

$x+1-1<2-1$

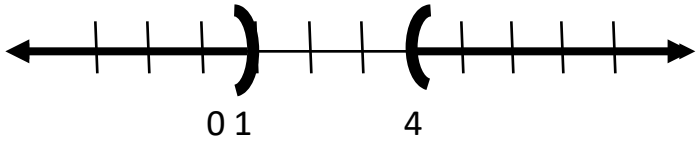
$x<1$



$x>4$



$x<1$ **or** $x>4$



* When two inequalities joined by "or", that means **union** of the solutions.

* Graph the inequalities separately.

* Look for everything shaded on the graph.

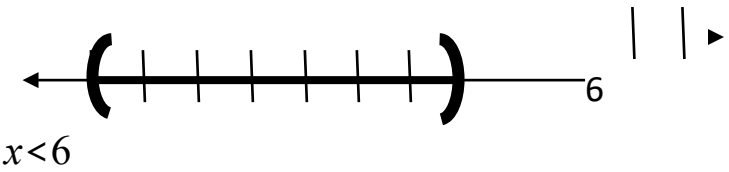
* What you see is what you get.

Write out the interval notation from

Interval Notation: $(-\infty,1)\cup(4,\infty)$ **Example (5):** $-5<x+3<9$

Solution: This is a **three-part** inequality. We will solve this inequality a little different than previous examples. However, our goal is to isolate the variable x in the middle.

$-5-3<x+3-3<9-3$



$-8<x<6$

-8 0

$(-8,6)$

*To isolate the variable x , we need to subtract 3 in the middle as well as two sides.

*State the solution in interval notation. (you can graph the solution on the number line to help you write out the interval notation.)

Example (6): $-2<7-3x\leq 19$

Solution: This is a **three-part** inequality, so our goal is to isolate the variable x in the middle. $-2-7<7-7-3x\leq 19-7$

$$-9<-3x\leq 12$$

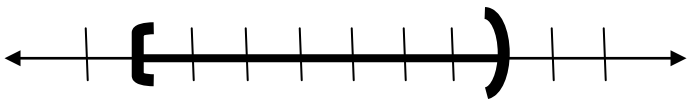
$$\frac{-9}{-3}>\frac{-3x}{-3}\geq\frac{12}{-3}$$

$$3>x\geq -4$$

*The first thing we need to do to isolate the variable x is subtracting 7 in the middle as well as two sides.

*Next we need to divide -3 in the middle as well as two sides and **Reverse** the inequality symbol.

* State the solution in interval notation. (you can graph the solution to help you write out the interval notation.)



-4 0 3

$$[-4,3)$$

Exercises: Solve the following inequalities. Write the solution in interval notation.

1. $2x+1\leq -1$ **or** $2x+1\geq 3$

2. $-1<5-2x\le 11$

3. $2t-3\ge 5t-(2t+1)$

4. $\frac{3x-2}{4}<\frac{2x+1}{5}$

5. $\frac{3}{2}(1-x)\le\frac{1}{4}-x$

Answers:

1. $(-\infty,-1]\cup[1,\infty)$ **2.** $[-3,3)$ **3.** $(-\infty,-2]$ **4.** $(-\infty,2)$ **5.** $\left[\frac{5}{2},\infty\right)$