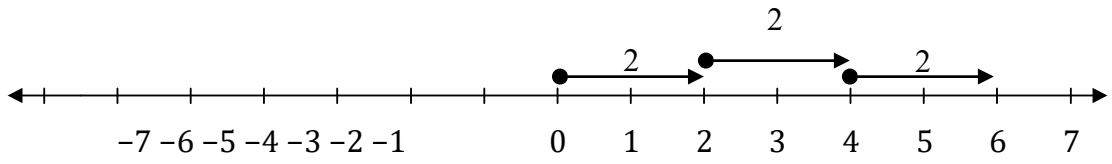
Multiplying and Dividing Integers

Multiplication is repeated addition. For example, $3\ 2\times$ means 3 groups of 2, or $2\ 2\ 2+\ +$.

This can be shown on a number line.

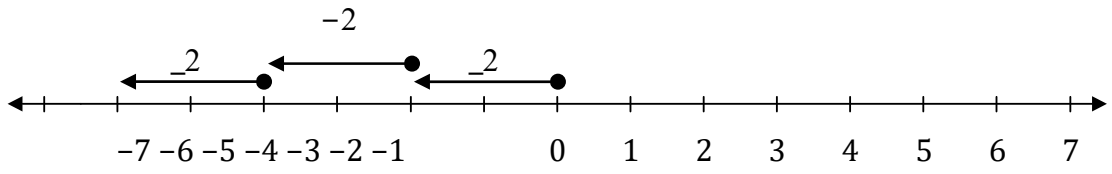


$3\ 2\ 6\times\ =$

We also know that we can change the order of the numbers we are multiplying so that $2\ 3\times$ or 2 groups of 3 also equal 6.

The number line can also be used to show multiplication of a positive and a negative number.

$3\ (\ 2)\times\ -$ means $-2 + (-2) + (-2)$, or 3 groups of negative 2.



$3\ (\ 2)\times\ -=\ -6$

It is not possible to show -2 groups of 3 on the number line, but because we can change the order in multiplication, $-\ 2\ 3\times$ is the same as $3\ (\ 2)\times\ -$, so $-\ 2\ 3\times\ =\ -6$.

<u>Same Signs:</u> When the signs of two factors are the same, both positive or both negative, multiply the numbers without worrying about the signs. The sign of the product (answer) will <u>always</u> be positive.

Examples:

$5 \cdot 7 = 35$ = ⟵ positive products ⟶

$(-5)(-7) = 35$

Both Positive Both negative

Different Signs: When the signs of two factors are different, one positive and one negative, multiply the numbers without worrying about the signs. The sign of the product (answer) will always be negative.

Example:

$5 \cdot (-7) = -35$ ⟵ negative product ⟶ $-5 \cdot 7 = -35$

one positive factor and one negative factor one negative factor and one positive factor

DIVISION of integers can be understood by realizing that a division cannot exist unless there is a related multiplication problem.

$\frac{8}{2} = 4$ because $4 \cdot 2 = 8$

$\frac{15}{3} = 5$ because $5 \cdot 3 = 15$

$\frac{24}{8} = 3$ because $3 \cdot 8 = 24$

NOTE that ***division by zero is undefined*** (not possible) because there is no related multiplication problem.

For example, we can not find a number n such that $\frac{8}{0} = n$ because $n \cdot 0 \neq 8$.

If we include some negative integers we see that there is still a related multiplication problem.

$\frac{-8}{2} = -4$ because $-4(2) = -8$

$\frac{15}{-3} = -5$ because $(-5)(-3) = 15$

$\frac{-24}{-8} = 3$ because $3(-8) = -24$

Same Signs: When the signs are the same, both positive or both negative, divide the numbers without worrying about the signs. The quotient (answer) will always be positive.

$\frac{18}{6} = 3$ ← positive quotient → $\frac{-18}{-6} = 3$

both positive — both negative

Different Signs: When the signs are different, one positive and one negative, divide the numbers without worrying about the signs. The quotient (answer) will always be negative.

$\frac{18}{-6} = -3$ ← negative quotient → $\frac{-18}{6} = -3$

one positive and one negative — one negative and one positive

EXERCISES: Multiply or divide.

1. $5(-4)$
2. $-3(-6)$
3. $-4 \cdot 9$
4. $(-2)(-10)$

5. $8 \cdot 6$ 6. $-14 \cdot 3$ 7. $(-6)(-9)$ 8. $(100)(5)$

9. $(-25)(-3)$ 10. $5(-9)$

11. $\frac{-14}{7}$ 12. $\frac{30}{-3}$ 13. $\frac{-10}{-2}$ 14. $\frac{0}{-8}$

15. $\frac{-45}{-5}$ 16. $\frac{48}{6}$ 17. $\frac{-12}{0}$ 18. $\frac{-4}{-4}$

19. $\frac{-27}{3}$ 20. $\frac{-105}{15}$ 21. $\frac{0}{0}$

KEY:

1.	−20	6.	−42	11.	−2	16.	8
2.	18	7.	54	12.	−10	17.	undefined
3.	−36	8.	500	13.	5	18.	1
4.	20	9.	75	14.	0	19.	−9
5.	48	10.	−45	15.	9	20.	−7
						21.	undefined