

Order of Operations Agreement

The Order of Operations Agreement tells us in what order we should perform operations when we have more than one operation in a single problem.

EXAMPLE: $18 \div (9 - 6) + (-3)^2$

This problem has a grouping symbol (parentheses), an exponent, a division, a subtraction inside the parentheses, and an addition. Without the Order of Operations Agreement to tell us what to do, it would be difficult to know where to start. The Order of Operations Agreement tells us to do things in the following order:

1. Simplify any grouping symbols first. Grouping symbols are parentheses (), brackets [], absolute value symbols | |, and a long fraction bar. A grouping symbol must encompass at least two numbers separated by an operational sign.

$$|-2 + 4| \quad \text{grouping symbol}$$

$$(9 - 6) \quad \text{grouping symbol}$$

$$(-3) \quad \text{not a grouping symbol}$$

$$\frac{3 + 7}{2} \quad \text{grouping symbol (long fraction bar)}$$

$$\frac{4}{2} \quad \text{not a grouping symbol}$$

2. Simplify exponential expressions: $(3)^2 = 9$
3. Perform all multiplications and divisions as you come to them going from left to right. This does not mean that you must multiply before you divide—you do them as they occur from left to right.

$$8 \cdot 3 \div 2 \quad \text{multiply first}$$

$$8 \div 2 \cdot 3 \quad \text{divide first}$$

4. Perform all additions and subtractions as you come to them going from left to right. This does NOT mean that you must add before you subtract—you do them as they occur.

$$8 + 7 - 3 \quad \text{add first}$$

$$8 \div 2 - 3 \quad \text{subtract first}$$

EXAMPLES:

Simplify: $18 \div (9 - 6) + (-3)^2$

$18 \div (9 - 6) + (-3)^2$	simplify inside the grouping symbol
$18 \div (3) + (-3)^2$	apply the exponent
$18 \div (3) + 9$	perform the division
$6 + 9$	perform the addition
15	the solution

Simplify: $-6 + |20 - 6| \div 7(-4)^2 - 3$

$-6 + 20 - 6 \div 7(-4)^2 - 3$	simplify inside the grouping symbol and the absolute value
$-6 + 14 \div 7(-4)^2 - 3$	simplify the exponent
$-6 + 14 \div 7(16) - 3$	perform the division
$-6 + 2(16) - 3$	perform the multiplication
$-6 + 32 - 3$	perform the addition
$26 - 3$	perform the subtraction
23	the solution

Sometimes we have grouping symbols INSIDE grouping symbols. When this happens, we start from the INSIDE and work our way out.

EXAMPLES:

Simplify: $4^2[16 - (7 - 1)] \div 10$

$4^2[16 - (7 - 1)] \div 10$	do inside parentheses
$4^2[16 - 6] \div 10$	Do inside the brackets
$4^2[10] \div 10$	simplify the exponent
$16[10] \div 10$	perform the multiplication
$160 \div 10$	perform the division

Simplify: $18 - 3 \cdot \frac{2(16)-12}{4+1} - (-3)$

Here our grouping symbol is a long fraction bar. We must follow the Order of Operations Agreement WITHIN the grouping symbol. We must also simplify the numerator and the denominator independently of each other.

$18 - 3 \cdot \frac{2(16)-12}{4+1} - (-3)$ simplify within grouping symbols
(numerator and denominator)

$18 - 3 \cdot \frac{32-12}{5} - (-3)$ continue to simplify the numerator

$18 - 3 \cdot \frac{20}{5} - (-3)$ reduce the fraction

$18 - 3 \cdot 4 - (-3)$ perform the multiplication

$18 - 12 - (-3)$ perform the subtraction

$6 - (-3)$ rewrite (if desired)

$6 + 3$ add

9 the result

This phrase may help you remember the sequence:

		<u>P</u> arentheses (all grouping symbols).....	<u>P</u> lease
		<u>E</u> xponents	<u>E</u> xcuse
Left to	}	<u>M</u> ultiplication	<u>M</u> y
Right		<u>D</u> ivision	<u>D</u> ear
Left to	}	<u>A</u> ddition	<u>A</u> unt
Right		<u>S</u> ubtraction	<u>S</u> ally

REMEMBER the PHRASE:

“Please Excuse My Dear Aunt Sally”

and it will help with the sequence.

EXERCISES: Simplify using the Order of Operations Agreement.

a. $12 - (12 - 4)^2 \div 4$

b. $10 + 1 - 5 \cdot 2 \div 5$

c. $2\{6 - 2[8 - (4 + 2)]\} - 1^3$

d. $4(3) - (5 - 2) \cdot 3^2$

e. $2[(16 \div 8) - (-2)] + 4^3$

f. $\frac{7 \cdot 5 - 8}{3(3)} + \frac{25 + 2}{5 + 4}$

g. $|3 - 9| - |18 - 2|$

KEY:

a. -4

c. 3

e. 72

g. -10

b. 9

d. -15

f. 6