

Order of Operations Agreement

The Order of Operations Agreement tells us in what order we should perform operations when we have more than one operation in a single problem.

EXAMPLE: $18 \cdot 9 \cdot 6 \div (-) (+ -3)^2$

This problem has a grouping symbol (parentheses), an exponent, a division, a subtraction inside the parentheses, and an addition. Without the Order of Operations Agreement to tell us what to do, it would be difficult to know where to start. The Order of Operations Agreement tells us to do things in the following order:

1. Simplify any grouping symbols first. Grouping symbols are parentheses (), brackets [], absolute value symbols | |, and a long fraction bar. A grouping symbol must encompass at least two numbers separated by an operational sign.

$|-2 + 4|$ grouping symbol

$(9 - 6)$ grouping symbol

$\frac{(-3)}{3 + 7}$ not a grouping symbol

$\frac{\quad}{2}$ grouping symbol (long fraction bar)

$\frac{4}{2}$ not a grouping symbol

2. Simplify exponential expressions: $(-3)^2 = 9$
3. Perform all multiplications and divisions as you come to them going from left to right. This does not mean that you must multiply before you divide—you do them as they occur from left to right.

$8 \cdot 3 \cdot 2 \cdot \div$ multiply first

$8 \cdot 2 \cdot 3 \div \cdot$ divide first

4. Perform all additions and subtractions as you come to them going from left to right. This does NOT mean that you must add before you subtract—you do them as they occur.

$8 \cdot 7 \cdot 3 +$ - add first

$8 \cdot 3 \cdot 7 - +$ subtract first

EXAMPLES:

Simplify: $18 \cdot 9 \cdot 6 \div (-) (+ -3)^2$

$$18 \div 6 \div (-) (+ -3)^2 \quad \text{simplify inside the grouping symbol}$$

$$18 \div 6 \div + - (3)^2 \quad \text{apply the exponent } 18 \div 6 \div + 3 \cdot 9$$

perform the division $6 \div 9$ perform the
addition 15 the solution

Simplify: $- + 6 \cdot 20 \cdot 6 \cdot 7 \cdot 4 - \div (-)^2 - 3$

$$- + 6 \cdot 20 \cdot 6 \cdot 7 \cdot 4 - \div (-)^2 - 3 \quad \text{simplify inside the grouping symbol and find the absolute value}$$

$$- + 6 \cdot 14 \cdot 7 \cdot 4 \div (-)^2 - 3 \quad \text{simplify the exponent}$$

$$- + 6 \cdot 14 \cdot 7 \cdot 16 \cdot 3 \div \cdot \quad - \quad \text{perform the division}$$

$$- + 6 \cdot 2 \cdot 16 \cdot 3 \cdot \quad - \quad \text{perform the multiplication}$$

$$- + 6 \cdot 32 \cdot 3 - \quad \text{perform the addition}$$

$$26 - 3 \quad \text{perform the subtraction}$$

$$23 \quad \text{the solution}$$

Sometimes we have grouping symbols INSIDE grouping symbols. When this happens we start from the INSIDE and work our way out.

EXAMPLES:

Simplify: $4 \cdot 16 \cdot 7 \cdot 1^2 \cdot [-(-)] \div 10$

$$4 \cdot 16 \cdot 7 \cdot 1^2 \cdot [-(-)] \div 10 \quad \text{do inside parentheses}$$

$$4 \cdot 16 \cdot 6 \cdot 10^2 \cdot [-] \div \quad \text{do inside brackets}$$

$$4 \cdot 10 \cdot 10^2 \cdot \div \quad \text{simplify the exponent}$$

$$16 \cdot 10 \cdot 10 \div \quad \text{perform the multiplication}$$

$$160 \div 10 \quad \text{perform the division}$$

$$16 \quad \text{the solution}$$

Simplify: $18 - 3 \cdot \frac{2(16) - 12}{4 + 1} - (-3)$

Here our grouping symbol is a long fraction bar. We must follow the Order of Operations Agreement WITHIN the grouping symbol. We must also simplify the numerator and the denominator independently of each other.

2 16()-12 simplify within grouping symbols $18-3 \cdot \frac{32-12}{5} - (-3)$

$18-3 \cdot \frac{32-12}{5} - (-3)$ continue to simplify the numerator

$18-3 \cdot \frac{20}{5} - (-3)$ reduce the fraction

$18-3 \cdot 4 - (-3)$ perform the multiplication

$18-12 - (-3)$ perform the subtraction

$6 - (-3)$ rewrite (if desired)

$6+3$ add

9 the result

This phrase may help you remember the sequence:

	<u>P</u> arentheses (all grouping symbols).....	<u>P</u> lease	
	<u>E</u> xponents	<u>E</u> xcuse	
Left to	}	<u>M</u> ultiplication	<u>M</u> y
Right		<u>D</u> ivision	<u>D</u> ear
Left to	}	<u>A</u> ddition	<u>A</u> unt
Right		<u>S</u> ubtraction	<u>S</u> ally

REMEMBER the PHRASE:

“Please Excuse My Dear Aunt Sally”

and it will help with the sequence.

EXERCISES: Simplify using the Order of Operations Agreement.

a. $12 - 4 - (-)^2 \div 4$

f. $12 - 4 - 2 - [4 - (-) + (3 - 5) - 8] - 1$

g. $6 - 8 - 12 - 15 - 20(-) \div +$

b. $10 + -1 - 5 - 2 - 5 \cdot \div$

c. $2 - 6 \{ -2 - 8 [- (4 + 2)] \} - 1^3$ h. $\frac{7 \cdot 5 - 8}{3(3)} + \frac{25 + 2}{5 + 4}$

$$d. 43(-523-)^2 \quad i. 2313\{[[-27(-3)]]\}+39(-11)$$

$$e. 2168[(\div)(-2)]+4^3 \quad j. 14831514-\frac{\quad}{\quad}+\frac{\quad}{\quad}-$$

$$k. 39|18|2--| \quad -$$

KEY:

- a. -4 c. 3 e. 72 g. -9 i. 24 k. -10
 b. 9 d. -15 f. -12 h. 6 j. -4