Adding and Subtracting Polynomials

ADDITION:

To add polynomials we must arrange the terms of the polynomials so that the like terms can be added together or combined. A vertical or horizontal format can be used. Most students find the vertical format easier to understand.

EXAMPLE: $(4x^2 + 7x - 8) + (-9x^2 - 3x + 10)$

In this example the parentheses are there to help you tell one polynomial from the other. Because the operation is addition, removing the parentheses will not change any of the values. We will now rewrite the polynomials in a vertical format being sure to write the like terms underneath each other.

REMEMBER that the sign in front of each term goes with it!
REMEMBER that like terms must have identical variable parts.

$(4x^2 + 7x - 8) + (-9x^2 - 3x + 10)$

$4x^2 + 7x - 8$

$-9x^2 - 3x + 10$

$-5x^2 + 4x + 2$

Now add the numerical coefficients and keep the same variable part.

EXAMPLE: $(3y^1 + 4y + 14) + (-4y^2 + 21)$

If you look carefully at this problem you will see that the constant terms are the only ones which can be combined.

$3y^2 + 4y + 14$

$^1\ y^3 - 4y^2 + 4y + 35$

You may find it easier to put in a zero as a place holder.

$^2\ y^3 + 0y^2 + 4y + 14$

$0y^3 - 4y^2 + 0y + 21$

^{2} 21 Now add the numerical coefficients.

$-4y$ +

SUBTRACTION:

In order to understand subtraction of polynomials we must go back to when we first learned to subtract integers. REMEMBER that we changed the operation of subtraction to addition and then changed the number we were subtracting to its opposite.

$$--8\ 14 = -8 + -(14) = -22$$

The same concept is used in subtraction of polynomials. We must change the operation of subtraction to addition and then change the second polynomial (the one we are subtracting) to its opposite.

To find the opposite of any polynomial, we must change EVERY term in the polynomial to its opposite.

$$-\left(8x^2 - 6x + 4\right) = -8x^2 + 6x - 4$$

EXAMPLE: $(12x^2 + 3x - 1) - (8x^2 - 6x + 4)$

$$(12x^2 + 3x - 1) + [-8x^2 + 6x - 4]$$

$$\begin{array}{r} 12x^2 + 3x - 1 \\ \underline{-8x^2 + 6x - 4} \\ 4x^2 + 9x - 5 \end{array}$$

EXAMPLE: $(2x^2 + 5x - 3) - (3x^3 + 2x - 5)$

$$(2x^2 + 5x - 3) + [-3x^3 - 2x + 5]$$

Now list like terms underneath each other and add. Be careful to keep the like terms together!

$$\begin{array}{rrr} & 2x^2 + 5x - 3 \\ -3x^3 & - 2x + 5 \\ \hline -3x^3 + 2x^2 + 3x + 2 \end{array}$$

EXERCISES:

ADD:

1. $(x^2 + x + 5) + (3x^2 - 10x + 4)$

2. $(-6x^2 + 7x + 3) + (3x^2 + x + 3)$

3. $(3y^3 + 4y + 9) + (2y^2 + 4y - 21)$

4. $(7x^3 + 4x - 1) + (2x^2 - 6x + 2)$

5. $(5a^3 - 6a^2 + 3a) + (a^2 - 2a - 3)$

SUBTRACT:

6. $(3x^2 + 2x - 2) - (5x^2 - 5x + 6)$

7. $(5y^2 - y + 2) - (-2y^3 + 3y - 3)$

8. $(-2x^2 - x + 4) - (-x^3 + 3x - 2)$

9. $(x^2 - 3xy) - (-2x^2 + xy)$

10. $(5y^2 - 2y + 1) - (-3y^2 - y - 2)$

KEY:

1. $4x^2 - 9x + 9$
2. $-3x^2 + 8x + 6$
3. $3y^3 + 2y^2 + 8y - 12$
4. $7x^3 + 2x^2 - 2x + 1$
5. $5a^3 - 5a^2 + a - 3$
6. $-2x^2 + 7x - 8$
7. $2y^3 + 5y^2 - 4y + 5$
8. $x^3 - 2x^2 - 4x + 6$
9.
10. $3x^2 - 4xy$

$8y^2 - y + 3$