Dividing a Polynomial by a Monomial

Polynomial fraction bar → $\dfrac{15x^3 - 10x^2 + 5x}{5x}$ ← represents the operation of division

Monomial → $5x$

<u>Important Ideas</u>

1. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

2. In each division, like bases are divided by subtracting the exponent in the denominator from the exponent in the numerator.

3. There must be the same number of terms in the **<u>quotient</u>** as there are in the original polynomial.

4. There will be some problems where the numerator and the denominator do not have a common factor.

<u>To Divide by Monomials</u>

1. Rewrite the division so that each term of the polynomial is divided by the monomial.

2. Divide the numerical coefficients.

3. Divide like bases by subtracting the exponents.

4. Rewrite any negative exponents in their equivalent forms with a positive exponent. This term will be a fraction.

5. If there is a factor in the denominator which is not also in the numerator, write that term as a fraction.

Example 1: Simplify: $\dfrac{15x^3 - 10x^2 + 5x}{5x}$

Rewrite the division: $\dfrac{15x^3}{5x} - \dfrac{10x^2}{5x} + \dfrac{5x}{5x}$

Divide the numerical

$= 3x^{3-1} - 2x^{2-1} + 1x^{1-1}$

coefficients and the $= 3x^2 - 2x + x^0 = 3x^2 - 2x + 1$

like bases

Note that the final term is "1"

REMEMBER that there must be the same number of terms in the quotient as there are in the original polynomial.

NOTE that $\frac{5x}{5x}$ is a number divided by itself which is always equal to "1."

Example 2: Simplify: $\frac{24y^5 + 16y^4 - 8y^3}{2y^2}$

$$\frac{24y^5}{2y^2} + \frac{16y^4}{2y^2} - \frac{8y^3}{2y^2}$$

Rewrite the division

$$= 12y^{5-2} + 8y^{4-2} - 4y^{3-2}$$

Divide the numerical coefficients and the like bases

$$= 12y^3 + 8y^2 - 4y$$

Example 3: Simplify: $\frac{12x^6 - 9x^4}{-3x}$

$$\frac{12x^6}{-3x} - \frac{9x^4}{-3x}$$

Rewrite the division

$$= -4x^{6-1} - -(3x^{4-1})$$

Divide the numerical coefficients and the like bases

$$= -4x^5 - -(3x^3)$$

$$= -4x^5 + 3x^3$$

NOTE that the denominator was negative in the above example. When this occurs special care must be taken with the signs.

$8x^4 + 4x^3 + 2x$

Example 4: Simplify:

$$\frac{\qquad}{2x^2}$$

Rewrite the division $\frac{8x^4}{2x^2}+\frac{4x^3}{2x^2}+\frac{2x}{2x^2}$

Divide the numerical coefficients and the like bases $= 4x^{4-2}+2x^{3-2}+1x^{1-2}$

$= 4x^2+2x^1+x^{-1}$

Rewrite the term with the negative exponent as a fraction $= 4x^2+2x^1+\frac{1}{x}$

NOTE that <u>**only**</u> the last term is a fraction. It is a common error to extend the fraction bar to the other terms as well.

Example 5: Simplify: $\frac{20x^4y^3+10x^2}{5xy^2}$

Rewrite the division $\frac{20x^4y^3}{5xy^2}+\frac{10x^2}{5xy^2}$

Divide the numerical coefficients and the like bases $=4x^{4-1}y^{3-2}+2x^{2-1}\cdot\frac{1}{y^2}$

Write the second term as a single fraction $= 4x^3y+\frac{2x}{y^2}$

NOTE that we could not divide by y^2 in the second term because there are no factors of y in the numerator.

<u>**Practice Exercises:**</u>

1. $\frac{x^3-x^2+x}{x}$

2. $\frac{6y^4-2y^2}{2y}$

3. $\dfrac{12x^4 + 6x^3}{-2x^2}$

4. $\dfrac{-2y^2 + 4y - 8}{2y}$

5. $\dfrac{-4x^5 - 3x^4 - x^3}{x^3}$

6. $\dfrac{16a\,b^3 \quad {}^2 - 4a\,b^2}{2ab}$

7. $\dfrac{18\,x\,y^5 \quad {}^3 + 15x\,y^4 \quad {}^4 - 12x \quad y^2 \quad {}^3}{3xy^2}$

8. $\dfrac{-3a^2 + 3a + 3}{-3}$

9. $\dfrac{24\,a\,b^{43} - 16a\,b \quad {}^3}{4a\,b_2}$

10. $\dfrac{3x\,y^2 \quad {}^3 - 3x \quad {}_2}{3y^2}$

Answers to Practice Exercises:

1. $x^2 - x + 1$ 5. $-4x^2 - 3x - 1$ 8. $a^2 - a - 1$

2. $3y^3 - y$ 6. $8ab^2 - 2ab^2$ 9. $_2 - 4a$

$6ab$ __ b

3. $-6x^2 - 3x$ 7. $6xy^4 + 5xy^3$ $^2 - 4xy$ 10. $_2 - x_2$

xy __ y

4. 4

$- + - y$

2

y