Multiplying Binomials With Special Products

In this assignment we will learn to multiply binomials which give special products. Recognition of these special products will become particularly important in further mathematics.

The first special product results from multiplying two binomials, one of which is the sum and the other the difference of two terms.

Here are some examples: $(a\ b\ a\ b+\ )(\ -\ )$

$(3x+2)(3x-2)$

$(y-7)(y+7)$

$(2a+5)(2a-5)$

Notice that with each pair of binomials the only thing that is different is the sign on the second term. In one binomial the second term is positive, giving us the *sum* of two terms, and in the other the second term is negative, giving us the *difference* of the same two terms.

We will now see what happens to each pair of binomials when we multiply them using the FOIL method.

$(a\ b\ a\ b+\ )(\ -\ )$

$a^2 - ab\ ab\ b+\ -\ ^2$

F O I L

$= a^2 + 0ab\ b-\ ^2$

$= a_2 - b_2$

$(3\ 2\ 3\ 2x+\ )(\ x-\ )$

$9x^2 - 6\ 6x+\ -x\ 4$

F O I L

$= 9x^2 +\ -0x\ 4 = 9x^2$

-4

$(y-7)(y+7)$

$y^2 + 7y - 7y - 49$

F O I L

$= y^2 + 0y - 49$

$= y^2 - 49$

$(2a+5)(2a-5)$

$4a^2 - 10a + 10a - 25$

F O I L

$= 4a^2 + 0a - 25$

$= 4a^2 - 25$

Notice that in each case the middle terms of the product (O & I) are opposites of each other and cancel each other out. This means that our final product will have only two terms. This product has a special name. It is called <u>the difference of two squares</u>. The only way to get a product which is the difference of two squares is to multiply two binomials which are the <u>sum and difference of two terms</u>.

> A short cut way to multiply the sum and difference of two terms is to recognize that the terms of the product are the squares of the first and second terms of the binomials.
>
> $$(A+B)(A-B) = A^2 - B^2$$

EXAMPLES:

$$(a+b)(a-b)$$

The square of the first term → $(a)^2 - (b)^2$ ← the square of the second term

$$a^2 - b^2$$

$$(3x+2)(3x-2)$$

The square of the first term → $(3x)^2 - (2)^2$ ← the square of the second term

$$9x^2 - 4$$

$$(y-7)(y+7)$$

The square of the first term $\rightarrow (y)^2 - (7)^2 \leftarrow$ the square of the second term

$$y^2 - 49$$

$$(2a+5)(2a-5)$$

The square of the first term $\rightarrow (2a)^2 - (5)^2 \leftarrow$ the square of the second term

$$4a^2 - 25$$

The sign between the two terms of the product is always negative because it comes from the "L" of FOIL. The "L" of FOIL is always the product of a positive and a negative number and consequently is always negative.

Another special product results from multiplying a binomial by itself, or *squaring* the binomial.

Here are some examples.

$$(a+b)(a+b)$$

$$(2y+3)^2 = (2y+3)(2y+3)$$

$$(4x-5)(4x-5)$$

$$(y-7)^2 = (y-7)(y-7)$$

NOTICE that the binomials in each pair are identical to each other.

We will now see what happens when we multiply each pair of binomials using the FOIL method.

$$(a+b)(a+b)$$

$$a^2 + ab + ab + b^2$$

F O I L

$$= a^2 + 2ab + b^2$$

$$(2y+3)^2 = (2y+3)(2y+3)$$

$$4y^2 + 6y + 6y + 9$$

F O I L

$$= 4y^2 + 2(6y) + 9$$

$$= 4y^2 + 12y + 9$$

$(4\ 5\ 4\ 5x-\ )(\ x-\ )$ $16x^2 -20\ 20\ 25x-\ x+$

F O I L

$=16x^2 + \ -2\ 20(x)+25$

$=16x^2 -40\ 25x+$

$(y-7)^2 = \ -(y\ 7)(y-7)$ y^2 -7y-7y+49

F O I L

$= y^2$-2(7y)+49

$=y^2$-14y+49

NOTICE the following things about these products.

a. **The third term is always positive.** This is because the third term comes from the "L" of FOIL. Because the binomials are both the same, the L of FOIL is the result of multiplying either two positive terms or two negative terms. In either case the product is always positive.

b. **The "O" and "I" of FOIL give two identical terms.** Consequently the middle term of the final product is *twice* one of those identical terms. Each of these terms is the product of the first and second term of one of the factors.

c. **The middle term of the product will have the same sign as that in the middle of the two factors.** The middle term of the product is positive if the terms of the binomials are positive. The middle term of the product is negative if the binomials have a negative term.

d. **The first term of the product is the square of the first term of the binomials.**

e. **The third term of the product is the square of the second term of the binomials.**

These special products are called Perfect Square Trinomials.

$$(A+B)^2 = \ +A^2\, 2AB\ B+\ ^2\ (A-B)^2 = \ -A^2\, 2AB\ B+\ ^2$$

Once we recognize that squaring a binomial gives a special product, we can take a short cut.

EXAMPLE 1: $(a\ b\ a\ b+\ )(\ +\ )$

The square of the first term →$(\ )a^2 + 2ab\ b + (\ )^2$ ← the square of the second term

↑— Twice the product of the first and second term

$= +a^2\, 2ab\ b + {}^2$

EXAMPLE 2: $(2\ 3y+\ )^2$

The square of the first term →$(2y)^2 + 2\ 6(\ y)\ (\ ) + \ 3^2$ ← the square of the second term

↑ Twice the product of the first and second term

$= 4y^2 + 12y + 9$

EXAMPLE 3: $(4\ 5\ 4\ 5x-\ )(\ x-\ )$

The square of the first term → $(4x)^2 + 2\ 20(-\ x)\ (+\ -5)^2$ ← the square of the second term

↑—Twice the product of the first and second term

$16x^2 - 40x + 25$

EXAMPLE 4: $(y-7)^2$

The square of the first term → $(\ )y^2 + 2\ 7(-\ y)\ (+\ -7)^2$ ← the square of the second term

↑_ Twice the product of the first and second term

$= -y^2\ 14y + 49$

EXERCISES:

a. $(x-4)(x+4)$ f. $(x-4)(x-4)$

b. $(2\ 9\ 2\ 9y+\ )(\ y-\ )$ g. $(2\ 9\ 2\ 9y+\ )(\ y+\ )$

c. $(a+6)(a+6)$ h. $(a+6)(a-6)$

d. $(3\ 4\ 3\ 4x-\ )(\ x+\ )$ i. $(3\ 4\ 3\ 4x-\ )(\ x-\ )$

e. $(y-2)^2$ j. $(2\ 1x+\ )_2$

KEY:

a.	x^2-16	f.	$x^2-8\ 16x+$
b.	$4y^2-81$	g.	$4y^2+36y+81$
c.	$a^2+12\ 36a+$	h.	a^2-36
d.	$9x^2-16$	i.	$9x^2-24\ 1x+\ 6$
e.	$y^2-\ +4y\ 4$	j.	$4x^2+4x+1$