Multiplying Monomials

In order to understand how to multiply monomials, you must understand what an exponent is telling you to do. REMEMBER that an exponent tells you how many times to multiply the base by itself.

x^2means $x \cdot x$ and x^4 means $x \cdot x \cdot x \cdot x$

If we multiply these together we will have $x\, x^{2} \cdot {}^{4}$

This will give us ($x\, x\, x\, x\, x\, x \cdot$)($\cdot$ $\cdot$ $\cdot$) which is x^6

A shorter way to do this would be to add the exponents together.

$$x \quad x_{2} \cdot {}_{4} = x_{2\,4+} = x_{6}$$

<u>The Rule for Multiplying Exponential Expressions</u> is as follows: If m and n (the exponents) are integers, then $x\, x^{m} \cdot {}^{n} = x^{m\,n+}$

This means that when you are multiplying two exponential expressions <u>with the same base</u>, you add the exponents and keep the base.

EXAMPLE: Multiply $y\, y^{5} \cdot {}^{3}$

The bases are the same so we add the exponents.

$$y \quad y_{5} \cdot {}_{3} = y_{5\,3+} = y_{8}$$

To multiply exponential expressions which involve numbers as well as variables, we follow these steps:

1. Use the Commutative and Associative Properties of Multiplication to change the order of the factors so that the numbers are together and the like bases are together.

2. Multiply the numbers.

3. Multiply like bases by adding the exponents.

EXAMPLE: Simplify: $(-4x^3y)(9x^4y^5)$

$(-4\cdot 9)(x^3 \cdot x^4)(y \cdot y^5)$ Rearrange factors (multiplication commutes)

$-36(x^{3+4})(y^{1+5})$ Remember that the exponent is 1 if there is none written.

$-36x^7y^6$

EXAMPLE: Simplify: $(2a^3)(-5a^2b^7c)(4bc^4)$

REMEMBER to follow the steps on the previous page.

$[2(-5)(4)](a^3 \cdot a^2)(b^7 \cdot b)(c \cdot c^4)$

$-40(a^{3+2})(b^{7+1})(c^{1+4})$

$-40a^5b^8c^5$

It is sometimes necessary to simplify powers of monomials. This means that we will have an exponential expression raised to a power.

$\left(x^2\right)^4$ means $x^2 \cdot x^2 \cdot x^2 \cdot x^2$

We now have the same base being multiplied by itself so we can add the exponents.

$x^2 \cdot x^2 \cdot x^2 \cdot x^2 = x^{2+2+2+2} = x^8$

A shorter way to do this is to multiply the exponents.

$\left(x^2\right)^4 = x^{2\cdot 4} = x^8$

<u>The Rule for Simplifying Powers of Exponential Expressions</u> is as follows:

If m and n (the exponents) are integers, then $\left(x^m\right)^n = x^{m\cdot n}$

This means that if we are raising a power to a power we multiply the exponents and keep the base.

EXAMPLE: Simplify: $\left(y^5\right)^3$

NOTICE that there are parentheses separating the exponents. This tells us that we are raising a power to a power and must multiply the exponents.

$$(y^5)^3 = y^{5\cdot 3} = y^{15}$$

EXAMPLE: Simplify: $(3^2)^3$

$$(3^2)^3 = 3^{2\cdot 3} = 3^6 = 729$$

NOTE that in the above problem we can simplify the problem by working out the exponential expression. This is possible because the base is a number and not a variable.

EXAMPLE: Simplify: $(-3^2)^3$

$$(-3^2)^3 = [(-1)\,3^2]^3 = -(1)^3\cdot 3^{2\cdot 3} = -1(3)^6 = -729$$

This type of problem often causes trouble. NOTICE that in the original problem the negative sign is not separated from the exponent by parentheses. This means that the part <u>inside</u> the parentheses is read as "the opposite of positive three squared." The base is <u>positive</u> three.

If you have particular difficulty with this type of problem, go to your instructor for help.

Sometimes we have more than one factor which is being raised to a power. These factors may be numbers or variables.

$$(3x^2y^4)^2 \text{ means } (3x^2y^4)\cdot(3x^2y^4)$$

We could now follow our earlier rules and group the numbers together and the like bases together and multiply.

$$(3\cdot 3)(x^2\cdot x^2)(y^4\cdot y^4)$$

$$(3\cdot 3)(x^{2+2})(y^{4+4}) = 9x^4y^8$$

A shorter way to do this is to raise each factor to the given power by multiplying each exponent inside the parentheses by the exponent on the outside.

$$(3x^2y^4)^2 = (3^{1\cdot 2})(x^{2\cdot 2})(y^{4\cdot 2}) = 3^2x^4y^8 = 9x^4y^8$$

<u>The Rule for Simplifying Powers of Products</u> is as follows:

If m, n and p (the exponents) are integers, then $(x\, y^{m} \cdot {}^{n})^{p} = x\, y^{m\,p\,n\,p\cdot}$

As we have already said this means that all of the exponents on the INSIDE must be multiplied by the exponent on the OUTSIDE. REMEMBER that if there is no exponent written, it is understood to be "1."

EXERCISES:
SIMPLIFY: KEY:

1. $x\, x^{3} \cdot {}^{5}$ 1. x^{8}

2. $m\, m\, m^{3} \cdot \quad {}^{7} \cdot$ 2. m^{11}

3. $(2x^{2})(-4x^{3})$ 3. $-8x^{5}$

4. $(-5ab^{3})(-3a^{6}b^{4})$ 4. $15a^{7}b^{7}$

5. $(x^{5})^{4}$ 5. x^{20}

6. $(-2^{3})^{2}$ 6. 64

7. $(5x^{2}y^{3})^{3}$ 7. $125x^{6}y^{9}$

8. $[(-2)^{2}mn^{8}]^{2}$ 8. $16m^{2}n^{16}$

9. $(-3xy)(2x^{2}y^{4})^{3}$ 9. $-24x^{7}y^{13}$ 10. $(3x)^{2}(3x^{3})^{2}$ 10. $81x^{8}$