

# Multiplying Polynomials

## Multiplying a Polynomial by a Monomial

To multiply a polynomial by a monomial we use a Distributive Property as well as the rule for multiplying exponential expressions.

EXAMPLE:  $4x^2(x + 8)$

We will first multiply  $4x^2$  and  $x$ . Then we will multiply  $4x^2$  and  $8$ .

$$4x^2(x) + 4x^2(8)$$

$$4x^3 + 32x^2$$

REMEMBER to add the exponents if the bases are the same.

$$4x^2(x) = 4x^{2+1} = 4x^3$$

Also, REMEMBER that the sign we get when we multiply gives us the sign between the terms.

EXAMPLE:  $-y(-3y^2 - 2y + 6)$

Use the Distributive Property to multiply each term inside the parentheses by  $-y$ . REMEMBER that the sign in front of the term goes with the term.

$$-y(-3y^2 - 2y + 6)$$

$$-y(-3y^2) - y(-2y) - y(6)$$

$$3y^3 + 2y^2 - 6y$$

REMEMBER that we cannot combine terms unless the variable parts are identical. This problem is simplified as far as possible. EXAMPLE:  $ab(2a^2 - 4ab - 6b^2)$

$$ab(2a^2) + ab(-4ab) + ab(-6b^2)$$

$$2a^3b - 4a^2b^2 - 6ab^3$$

Don't forget the rules for exponents!

## Multiplying a Polynomial by a Polynomial

Multiplication of polynomials can be accomplished by using a horizontal format and the Distributive Property, or by using a vertical format. We will use the vertical format—the process is similar to multiplication of real numbers.

EXAMPLE:  $(y^2 - 2y + 7)(y - 2)$

REWRITE in vertical format.  $y^2 - 2y + 7$   $y - 2$

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Multiply each term by  $-2$ :

$$\begin{array}{r} y^2 - 2y + 7 \\ \times y - 2 \\ \hline -2y^2 + 4y - 14 \end{array}$$

Now multiply each term by  $y$ . Be sure to keep like terms lined up.

$$\begin{array}{r} y^2 - 2y + 7 \\ \times y - 2 \\ \hline -2y^2 + 4y - 14 \\ y^3 - 2y^2 + 7y - 14 \\ \hline y^3 - 4y^2 + 11y - 14 \end{array}$$

Now combine like terms

If you compare multiplication of polynomials to long multiplication of integers you will see that the steps are very similar.

$$\begin{array}{r} 379 \\ \times 23 \\ \hline 1137 \end{array}$$

Multiply by 3    Multiply by  $-2$

379

$$\begin{array}{r} 3x^2 + 35x - 5 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3x^2 + 35x - 5 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4x - 2 \\ \times -6x^2 - 6x + 10 \\ \hline \end{array}$$

$\times 23$  Multiply by 2, keeping like terms together  
 4x, keeping like terms together  
 1137  
 758  
 379

$$\begin{array}{r} 4x-2 \\ -6x^2-6x+10 \end{array}$$

$$12x^3 + 12x^2 - 20x$$

$$3x^2 + 3x - 5$$

$$\begin{array}{r} 4x-2 \\ -6x^2-6x+10 \end{array}$$

$\times 23$

Now add to get the like total.  $12x^3 + 12x^2 - 20x$

Add by combining like terms. Sometimes there is a missing term in one of the polynomials. This means we must either leave a space or put in a place holder. We will use a place holder.

EXAMPLE:  $(3a^3 - 5a^2 + 7)(6a - 1)$

Notice that the first polynomial has no "a" term. We'll hold that place with "0a."

$$\begin{array}{r} 3a^3 - 5a^2 + 0a + 7 \\ 6a - 1 \end{array}$$

$$-3a^3 + 5a^2 - 0a + 7 \leftarrow \text{multiply by } -1$$

$$18a^4 - 30a^3 + 0a^2 + 42a \leftarrow \text{multiply by } 6a$$

$$18a^4 - 33a^3 + 5a^2 + 42a - 7 \leftarrow \text{add}$$

EXAMPLE:  $(5x^3 - 5x + 2)(x - 4)$

There is no "x<sup>2</sup>" term in the first polynomial.

$$5x^3 + 0x^2 - 5x + 2$$

$$-20x^3 + 0x^2 + 20x - 8 \leftarrow \text{multiply by } -4$$

$$5x^4 + 0x^3 - 5x^2 + 2x \leftarrow \text{multiply by } x$$

$$5x^4 - 15x^3 + 6x^2 - 26x + 10$$

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$$5x^4 - 20x^3 - 5x^2 + 22x - 8 \leftarrow \text{add}$$

EXERCISES: Multiply each of the following.

a.  $-6a^3(a + 2)$       f.  $(-x^2 + 3x - 2)(2x - 1)$

b.  $2y(3y^2 - 4)$       g.  $(3y^2 + 2y - 2)(5y - 4)$

c.  $-x(-2x^4 - 3x^2 + 2)$       h.  $(-2x^2 + 3x + 8)(x + 7)$

d.  $(3y^3 - 2y^2 + 5)4y$       i.  $(4a^3 - 2a + 5)(a + 6)$

e.  $-3ab(a^2 - 5ab + 3b^2)$       j.  $(5y^3 + 2y^2 - 9)(2y - 3)$

KEY:

a.  $-6a^4 - 12a^3$

b.  $6y^3 - 8y$

c.  $2x^5 + 3x^3 - 2x$

d.  $12y^4 - 8y^3 + 20y$

e.  $-3a^3b + 15a^2b^2 - 9ab^3$

f.  $-2x^3 + 7x^2 - 7x + 2$

g.  $15y^3 - 2y^2 - 18y + 8$

h.  $-2x^3 - 11x^2 + 29x + 56$

i.  $4a^4 + 24a^3 - 2a^2 - 7a + 30$

j.  $10y^4 - 11y^3 - 6y^2 - 18y + 27$