The Quadratic Formula

Using the q**uadratic formula, we** can solve all quadratic equations.

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve the equations $6x - 1 = x^2$

First we put the equation in **standard form** by subtracting x^2 from each side.

$-x^2 + 6x - 1 = 0$

We will use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$**, where** $a = -1$, $b = 6$, $c = -1$**.**

$\dfrac{-6 \pm \sqrt{(6)^2 - (4)(-1)(-1)}}{2(-1)}$ Substitute $\boldsymbol{a = -1, b = 6, c = -1}$ into the formula. Place the parentheses on the numbers to avoid making mistakes on "signs"

$\dfrac{-6 \pm \sqrt{36 - 4}}{-2}$ Simplify.

$\dfrac{-6 \pm \sqrt{32}}{-2}$ Simplify the radical part, using the fact that $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$.

$\dfrac{-6 \pm 4\sqrt{2}}{-2}$ or $\dfrac{-6}{-2} \pm \dfrac{4\sqrt{2}}{-2}$ Factor the numerator (the -2 is a factor of both terms in numerator).

$\dfrac{-2(3 \pm 2\sqrt{2})}{-2}$ Cancel the common factor of -2 from the numerator and denominator.

$3 \pm 2\sqrt{2}$ There are two distinct solutions.

$3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$

Note: the fact that $b^2 - 4ac$ is not equal to a perfect square indicates that it is not possible to solve this equation by factoring.

Exercises: Solve the equations using quadratic formula.

1. $x^2 + 2x - 24 = 0$ **2.** $2x(x - 3) = 2$ **3.** $\frac{1}{2}x^2 + \frac{3}{2}x - 2 = 0$ **4.** $7x^2 + 4 = 2x$

Answers:

1. $\{4, -6\}$ **2.** $\left\{\frac{3 \pm \sqrt{13}}{2}\right\}$ **3.** $\{-4, 1\}$ **4.** $\left\{\frac{1 \pm 3\sqrt{3}\,i}{7}\right\}$