# Adding and Subtracting Square Roots

Previously, you learned that it is possible to add or subtract like terms using the distributive property. Similarly, it is possible to use the distributive property to add or subtract like square root expressions. Like square roots are those which have the same radicand. Unlike square roots, those which have different radicands when expressed in simplest form, cannot be added or subtracted.

| Like Square Roots | Unlike Square Roots |
|---|---|
| $7\sqrt{3}$ and $5\sqrt{3}$ | $3\sqrt{5}$ and $2\sqrt{7}$ |
| $8\sqrt{x}$ and $-2\sqrt{x}$ | $7\sqrt{y}$ and $9\sqrt{x}$ |
| $10\sqrt{x+1}$ and $3\sqrt{x+1}$ | $5\sqrt{x-1}$ and $5\sqrt{x+1}$ |

There are restrictions placed on those radical expressions which contain variables under the radical symbol. Expressions for which the value of the variable will give a radicand that is less than 0 (i.e. negative) will not be considered in MAT0024. Radical expressions in which the radicand is negative will be discussed in MAT1033 and subsequent math courses.

The like square roots above can be given in expressions and combined using the distributive property as follows.

1. $7\sqrt{3} + 5\sqrt{3} = (7+5)\sqrt{3} = 12\sqrt{3}$

2. $8\sqrt{x} - 2\sqrt{x} = (8-2)\sqrt{x} = 6\sqrt{x}$

3. $10\sqrt{x+1} + 3\sqrt{x+1} = (10+3)\sqrt{x+1} = 13\sqrt{x+1}$

Note how easy it is to complete the process mentally.

1. Determine whether the square roots are like.
2. If the square roots are like, add or subtract the numbers outside the radical symbol. (Remember, if no numerical or variable factors are outside the radical symbol, the factor outside, sometimes referred to as the coefficient, is understood to be 1.)
3. Multiply the sum or difference that you obtain in step 2 by the radical expression to obtain your final answer.

Sometimes, unlike square root expression can be simplified and changed into like square roots; once simplified, those that are like can be combined.

The following examples are unlike square roots that can be simplified and subsequently added or subtracted. (Please do not use the decimal approximation obtained when using your calculators for radical that are not perfect squares. Be sure to give all answers in simple radical form.)

EXAMPLE 1:

$$\sqrt{8}+\sqrt{50}=\sqrt{4\cdot 2}+\sqrt{25\cdot 2}$$
$$=2\sqrt{2}+5\sqrt{2}$$
$$=7\sqrt{2}$$

EXAMPLE 2:

$$x\sqrt{x^3}-5\sqrt{x^5}=x\sqrt{x^2\cdot x}-5\sqrt{x^4\cdot x}$$
$$=x\cdot x\sqrt{x}-5x^2\sqrt{x}$$
$$=-4x^2\sqrt{x}$$

EXAMPLE 3:

$$\sqrt{250x^2y^3}+x\sqrt{40y^3}=\sqrt{25\cdot 10\,x^2y^2\cdot y}+x\sqrt{4\cdot 10\,y^2\cdot y}$$
$$=5xy\sqrt{10y}+2xy\sqrt{10y}$$
$$=7xy\sqrt{10y}$$

EXAMPLE 4:

$$7x\sqrt{16x^3}-5\sqrt{25x^5}=7x\sqrt{16x^2\cdot x}-5\sqrt{25x^4\cdot x}$$
$$=7\cdot 4x\cdot x\sqrt{x}-5\cdot 5\cdot x^2\sqrt{x}$$
$$=28x^2\sqrt{x}-25x^2\sqrt{x}$$
$$=3x^2\sqrt{x}$$

The following examples represent unlike square roots that can be simplified but are not like radicals, therefore, they cannot be added or subtracted.

EXAMPLE 5:

$$\sqrt{27}+\sqrt{180}$$
$$=\sqrt{9\cdot 3}+\sqrt{36\cdot 5}$$
$$=3\sqrt{3}+6\sqrt{5}$$

EXAMPLE 6:

$$\sqrt{90}-\sqrt{200}$$
$$=\sqrt{9\cdot 10}-\sqrt{100\cdot 2}$$
$$=3\sqrt{10}-10\sqrt{2}$$

EXAMPLE 7:

$$x\,y^3+y^3$$

EXAMPLE 8:

$$3\;18a\;b\;c^{4\;5}+3\;125a\;b\;c^{2\;3\;2}$$

$$= \sqrt{x^2 xy} + \sqrt{y^2 y}$$
$$= x\sqrt{xy} + y\sqrt{y}$$

$$= 3\sqrt{9 \cdot 2 \cdot a^4 b^4 bc} + 3\sqrt{25 \cdot 5 \cdot a^2 b^2 bc}$$
$$= 3 \cdot 3 \cdot a^2 b^2 \sqrt{2bc} + 3 \cdot 5 \cdot ab\sqrt{5bc}$$
$$= 9a^2 b^2 \sqrt{2bc} + 15ab\sqrt{5bc}$$

Recall that radical expressions such as we have here are real numbers. As such, all of the properties of real numbers hold true when simplifying radical expressions. Note how like square roots in the following expressions use the commutative and associative properties to combine like terms. As before, this process can be completed mentally.

EXAMPLE 9:

$$7\sqrt{2} + 5\sqrt{3} - \sqrt{2} + 3\sqrt{3}$$
$$= \left(7\sqrt{2} - \sqrt{2}\right) + \left(5\sqrt{3} + 3\sqrt{3}\right)$$
$$= 6\sqrt{2} + 8\sqrt{3}$$

EXAMPLE 10:

$$5y\sqrt{y^2} - 4\sqrt{x^3} + y^2 - 7x\sqrt{x^2}$$
$$= 5y \cdot y - 4x\sqrt{x} + y^2 - 7x \cdot x =$$
$$6y^2 - 4x\sqrt{x} - 7x^2$$

**EXERCISES**

Simplify the following:

1. $8\sqrt{2} + 7\sqrt{2}$

2. $4\sqrt{3} - 6\sqrt{3}$

3. $15\sqrt{y} + \sqrt{y}$

4. $3x\sqrt{5} - x\sqrt{5}$

5. $7\sqrt{8} - 4\sqrt{32} + 6\sqrt{50}$

6. $2\sqrt{x^2} - 9x + 7\sqrt{x^2}$

7. $11\sqrt{96} + \sqrt{150} - 8\sqrt{18}$

8. $\sqrt{3x^3} + 3\sqrt{12x^3} - x\sqrt{75}$

9. $\sqrt{-121x^2} - \sqrt{144x^3} + x\sqrt{225x}$

10. $3xy^2\sqrt{y^2} - 10xy^{2\,2}$

Answers:

1. $15\sqrt{2}$
2. $-2\sqrt{3}$
3. $16\sqrt{y}$
4. $2x\sqrt{5}$
5. $28\sqrt{2}$
6. $0$
7. $49\sqrt{6} - 24\sqrt{2}$
8. $3x\sqrt{x} + 6x\sqrt{x3} \quad -5x\sqrt{3}$
9. $-11x + 3x\sqrt{x}$
10. $-7xy^{2\,2}$