Solving Radical Equations

Radical equations are equations contain radical expressions. The radical equations we are going to solve are mainly square root equations and cubic root equations.

Example #1: Solve $\sqrt{x}=8$

Solution:

The first thing we need to do to solve radical equations is to remove the radical (*n*th roots).

$\sqrt{x}=8$ — To remove the square root on the left side, we will need to square both sides of the equation.

$\left(\sqrt{x}\right)^2=\left(8\right)^2$ — Simplify each side of the equation.

$x=64$

$\sqrt{64}=8 \quad 8=8 \checkmark$ Check the answer. $x=64$ is the solution.

Example #2: Solve $\sqrt{2x-5}=3$ Solution:

This equation looks a little different than the previous one. The **radicand** (the expression under the radical sign) of the previous equation is x. The radicand of this equation is $2x-5$. But, as long as the **radical term** is isolate, we can follow the same steps to solve the equation as mentioned above.

$\sqrt{2x-5}=3$ — To remove the square root on the left side, we will need to square both sides of the equation.

$\left(\sqrt{2x-5}\right)^2=\left(3\right)^2$ — Simplify each side of the equation.

Solve for x.

$2x-5=9 \quad 2x=14$

$x=7$

$\sqrt{2\left(7\right)-5}=3$ — Check the answer.

x

$\sqrt{9}\ 3=\quad 3\ 3=$ ✓ $=7$ is the solution.

Example #3: Solve $\sqrt{2x+8}=x$ Solution:

$\sqrt{2x+8}=x$ To remove the square root on the left side, we will need to square both sides of the equation.

$\left(\sqrt{2x+8}\right)^2=(x)^2$ Simplify each side of the equation.

Solve for x.

$2x+8=x^2$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

To solve a quadratic equation, we need to set one side of the equation equal to zero. Then factor the equation.

$x=4$ or $x=-2$

$\sqrt{2(4)+8}=4$ We have to check the solutions to see if they work.

$\sqrt{16}=4$

$4=4$ ✓ When substitute 4 into the equation, we receive a true statement. Therefore 4 is a solution.

$\sqrt{2(-2)+8}=-2$

$\sqrt{4}=-2$ When substitute -2 into the equation, the result is not a true statement. So -2 is not a

$2=-2$ X solution.

$x=4$ is the solution

Example #4: Solve $\sqrt{4-x}+5=8$

Solution:

The radical term in this equation $\sqrt{4-x}$ is not isolated (not by itself). So we have to isolate (remove +5) the radical term before we can follow the same steps to solve the equation as mentioned above.

$\sqrt{4-x}+5=8$ To remove +5 on the left side, we will need to subtract 5 on both sides of the equation.

$\sqrt{4-x}+5-5=8-5$

$\sqrt{4-x}=3$ Now the radical is isolated. To remove the square root on the left side, we will need to square both sides of the equation.

$\left(\sqrt{4-x}\right)^2=(3)^2$

$4-x=9$ Simplify each side of the equation.

$x=-5$ Solve for x.

$\sqrt{4-(-5)}=3$ We have to check the solution to see if it works.

$\sqrt{9}=3$ $x=-5$ is the solution.

$3=3$ ✓

Example #5: Solve $\sqrt{4-y}=y-2$

Solution:

$\left(\sqrt{4-y}\right)^2=(y-2)^2$ The radical is isolated. We will need to square both sides of the equation to remove the square root on the left side.

$4-y=(y-2)(y-2)$

$4-y=y^2-4y+4$ We need to FOIL the right side and simplify the equation.

$y^2-3y=0,\ y(y-3)=0\ \ y=0$ or $y=3$ Solve for y.

$\sqrt{4-0}=0-2$ We have to check the solution to see if it works.

$\sqrt{4}=-2$, $2\neq-2$ ✗

$\sqrt{4-3}=3-2$ $y=3$ is the solution.

$\sqrt{1}=1$, $1=1$ ✓

Example #6: Solve $\sqrt[3]{x} = -4$ Solution:

The first thing we need to do to solve this radical equation is to remove the radical (*n*th roots).

$\left(\sqrt[3]{x}\right)^3 = (-4)^3$ The radical is isolated. We will need to cube both sides of

$x = -64$ the equation to remove the cubic root on the left side.

✓ We have to check the solution to see if it works.

$\sqrt[3]{-64} = -4$ $x = -64$ is the solution.

Example #7: Solve $\sqrt[3]{x+10} = 4$ Solution:

$\sqrt[3]{x+10} = 4$ The radical is isolated. We will need to **cube** both sides

$\left(\sqrt[3]{x+10}\right)^3 = (4)^3$ of the equation to remove the cubic root on the left side.

$x + 10 = 64$ $x = 54$

✓ We have to check the solution to see if it works.

$\sqrt[3]{54+10} = \sqrt[3]{64} = 4$ $x = 54$ is the solution.

Exercises: Solve the following radical equations

1. $\sqrt{3y-1} = 5$ **2.** $\sqrt[3]{x-2} = -4$ **3.** $x - 1 = \sqrt{5x-9}$ **4.** $\sqrt{d} + d = 6$ **5.** $x = \sqrt{x+7} - 1$

Answers:

1. $\{\ \}12$ **2.** $\{-4\}$ **3.** $\{5,2\}$ **4.** $\{\ \}9$ **5.** $\{\ \}10$