Solving Equations in the Form ax = b

In equations of the form $ax = b$ (a times x equals b), x is a variable which represents an unknown number and a and b are constants.

EXAMPLES: $ax\ b =$

$$3x = 12$$

$$-4y = -16$$

$$x = 9$$

To solve an equation we must find the value of the variable so that the original equation is true when the variable is replaced with that value.

EXAMPLE: $3\ 12x =$

If x is replaced with 4, the equation is true.

$$3x = 12$$

$$3(4)\ 12 =$$

$$12 = 12 \text{ TRUE}$$

To solve these equations we must use the Multiplication Property of Equations. **NOTE** that the final goal in solving the equation is to have a statement where the variable is equal to the constant. The solution is the constant.

SOLVE: $5x = 75$

To get x by itself on one side of the equation we must change the coefficient of x from 5 to 1. We will do this by multiplying **<u>both sides</u>** of the equation by the **<u>reciprocal</u>** of 5.

$$5x = 75$$

$$\frac{1}{5} \times 5x = 75 \times$$

$$\frac{15}{55}x = \frac{75}{5}$$

Reduce the fractions and we have:

$$1x = 15$$

Multiplying a number by one does not change the number.

$1x = 15$ is the same as $x = 15$

CHECK:

$$5x = 75$$
$$5(15) = 75$$
$$75 = 75$$
TRUE

EXAMPLE: Solve: $\frac{4x}{5} = 16$

4 times x, divided by 5 is the same as $\frac{4}{5}$ times x.

The first thing we will do is rewrite $\frac{4x}{5}$ as $\frac{4}{5} \cdot x$

Multiply both sides by the reciprocal of $\frac{4}{5}$

$$\frac{5}{4} \cdot \frac{4}{5} \times x = \frac{16}{1} \cdot \frac{5}{4} \times$$

$$\frac{20}{20}x = \frac{80}{4}$$

$$1x = 20$$
$$x = 20$$

CHECK:

$$\frac{4}{5}x = 16$$
$$\frac{4(20)}{5} = 16$$
$$\frac{80}{5} = 16$$
$$16 = 16$$
TRUE

In some problems it is necessary to combine like terms before solving the equation.

EXAMPLE: $8y - 6y = 14$

$8y$ and $6y$ are like terms on the same side of the equals sign. We must combine variable terms so that there is only one variable term before we begin to solve.

$$8y - 6y = 14 \quad \text{Combine like terms}$$

$$2y = 14$$

$$\frac{1}{2} \times 2y = 14 \times \frac{1}{2} \quad \text{Multiply both sides by the reciprocal of 2}$$

$$1y = \frac{14}{2} \quad y = 7$$

Simplify

NOTE: Be very careful when the coefficient is negative and remember that the reciprocal of a negative number is also negative.

Negative coefficient

$$-4x = 12$$

$$\frac{1}{-4} \cdot (-4)x = \frac{1}{-4} \cdot 12$$

$$x = -3$$

EXERCISES: Solve and check.

1. $-3x = 18$
2. $\frac{4}{9}x = 12$
3. $7y = 21$
4. $-32 = 8n$
5. $-12x = -144$
6. $\frac{x}{3} = 15$
7. $\frac{2n}{3} = 2$
8. $5x + 3x = 24$
9. $2n - 6n = 28$
10. $-\frac{2}{5}x = -\frac{5}{8}$

KEY:

1. $x = -6$
6. $x = 45$

2. $x = 27$ 7. $n = 3$

3. $y = 3$ 8. $x = 3$

4. $n = -4$ 9. $n = -7$

5. $x = 12$ 10. $x = \frac{25}{16}$