# Solving Equations in the Form x + a = b

To *solve* an equation means to find the value of the variable so that the original equation is true when the variable is replaced with the value.

**EXAMPLE:** $x + 3 = 8$

If $x$ is replaced with 5, the equation is true.

$$x + 3 = 8$$
$$\downarrow$$
$$5 + 3 = 8$$
$$8 = 8 \text{ which is true}$$

To solve equations, we will use the following properties:

**Addition Property of Equations**.
The same number can be added to each side of an equation without changing the solution.

If $a$ = b, then $a + c = b + c$ and the solution stays the same.

**Addition Property of Opposites**
The sum of a term and its opposite is zero.

$$5 + (-5) = 0$$
$$-4 + 4 \quad = 0$$

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2 ( 2)| 0

3 |(−3)= +

$$a + (-a) = 0$$

**Addition Property of Zero**
The sum of a term and zero is the term

$$5 + 0 = 5$$
$$0 + (-4) = -4$$
$$a + 0 = a$$

In equations of the form $x + a = b$, $x$ is a variable which represents an unknown number and $a$ and $b$ are constants.

**EXAMPLES:** $\underline{x + \ a = b}$

$$x + 3 = 8\ x$$
$$-\ 5 = -6$$

NOTE that $x- \ = -5 \quad 6$still fits the form $x + a = b$, though the operation is subtraction and not addition. Remember that subtraction can be rewritten as addition of the opposite.

$$\underline{x + a = b}$$
$$x - 5 = -6$$
$$\downarrow \quad \downarrow$$
$$x + (-5) = -6$$

Our final goal in solving an equation is to have a statement where the variable is equal to the constant. The solution to the equation is the constant.

**SOLVE:** $x \quad + \quad 12 \quad = \quad -4$

To get $x$ by itself on one side of the equation we must remove 12 from the left side of the equation. To do this we will add the **<u>opposite of</u>** 12 to both sides of the equation.

$$x \quad + \quad 12 \qquad = \quad -4 \qquad x$$
$$+ \quad \underbrace{12 + (-12)} = \ \underbrace{-4 + (-12)}$$

Now we will combine like terms: $x \ + \ 0 \ = \ -16$

Zero added to any number is the number itself, so $x + 0 = -16$ is the same thing as $x = -16$.
To check we will replace $x$ with $(-16)$ in the original equation. $x$

$$+ 12 = -4$$
$$\downarrow$$
$$\underbrace{(-16) + 12} = -4$$

$$-4 = -4 \quad \text{TRUE}$$

Be sure you understand each step. Get help if you don't understand.

**SOLVE:** $x - 4 = -6$

Since $x - 4$ is equivalent to $x + (-4)$, you do not
change the other side. Try to do this first step mentally!

$$x + (-4) = -6$$

Add the opposite of $-4$ to both sides.

$$x + (-4) + 4 = -6 +$$
$$4\ x + 0 = -2\ x = -2$$

CHECK: $-2 - 4 = -6$

**EXAMPLE:**

$$x - \frac{3}{8} = \frac{1}{2}$$

$$x - \frac{3}{8} + \frac{3}{8} = \frac{1}{2} + \frac{3}{8}$$

Add the opposite of $\left(-\frac{3}{8}\right)$ to both sides.

$$x - 0 = \frac{1}{2} + \frac{3}{8}$$

$$x = \frac{4}{8} + \frac{3}{8}$$

Recall that to add fractions you **MUST** have a common denominator! The LCD is 8, so

$$\frac{1}{2} + \frac{3}{8} = \frac{1}{2} \cdot \frac{4}{4} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8}$$

$$x = \frac{7}{8}$$

CHECK:

$$x - \frac{3}{8} = \frac{1}{2}$$

$$\frac{7}{8} - \frac{3}{8} = \frac{1}{2}$$

$$\frac{4}{8} = \frac{1}{2} \quad \text{reduce the fraction} \rightarrow$$

$$\frac{1}{2} = \frac{1}{2} \qquad \text{TRUE}$$

**NOTE** that your goal is still to get *x* by itself by adding the opposite of the constant term to both sides.

**EXAMPLE:**

$$-5 = 9 + x$$

$$-5 + (-9) = 9 + (-9) + x$$

$$-14 = x$$

CHECK:

$$-5 = 9 + x$$

$$\downarrow$$
$$-5 = 9 + (-14)$$
$$-5 = -5 \qquad \text{TRUE}$$

**EXERCISES: Solve and check.**

1. $x - 4 = 11$ 2. $m + 9 = 2$ 3. $x + 7 = 7$ 4. $2 = x + 7$

5. $9 + a = -3$ 6. $y + \frac{3}{4} = -\frac{1}{4}$ 7. $x + \frac{1}{6} = -\frac{1}{3}$ 8. $\frac{4}{9} + a = -\frac{2}{9}$

9. $13 = -6 + m$ 10. $4 = -10 + y$

**KEY:**

1. $x = 15$ 2. $m = -7$ 3. $x = 0$ 4. $x = -5$ 5. $a = -12$

6. $y = -1$ 7. $x = -\frac{1}{2}$ 8. $x = -\frac{2}{3}$ 9. $m = 19$ 10. $y = 14$