

Solving Literal Equations

A **Literal Equation** is an equation containing more than one variable. We can solve a literal equation for any one variable in terms of the others. For example, if we wish to solve $x - y = b$ for x , we will need to add y to each side of the equation in order to isolate x : $x - y = b$

$$\begin{aligned}x - y + y &= b + y \\x &= b + y\end{aligned}$$

Example: Solve $AC = V$ for A . Divide both sides of the equation by C in order to isolate A :

$$\frac{AC}{C} = \frac{V}{C} \quad \text{Cancel the } C\text{'s on the left side of the equal sign.}$$

$$A = \frac{V}{C}$$

Example: Solve $2x + y = 5$ for y :

$$\begin{aligned}2x + y &= 5 \\2x - 2x + y &= 5 - 2x \\y &= 5 - 2x\end{aligned}$$

Example: Solve $2x + 3y = 6$ for y :

$$\begin{aligned}2x + 3y &= 6 \\2x - 2x + 3y &= 6 - 2x \\3y &= 6 - 2x \\\frac{3y}{3} &= \frac{6 - 2x}{3} \\y &= \frac{6 - 2x}{3}\end{aligned}$$

Note: This answer could also be written as

$$\begin{aligned}y &= -\frac{2x}{3} + \frac{6}{3} \text{ or} \\y &= 2 - \frac{2x}{3}\end{aligned}$$

Example: Solve $4(2x - 3b) = 7x + 5b$ for x :

$$\begin{aligned}4(2x - 3b) &= 7x + 5b \\8x - 12b &= 7x + 5b\end{aligned}$$

$$\begin{aligned}
 8x - 7x - 12b &= 7x - 7x + \\
 5bx - 12b &= 5b \\
 x - 12b + 12b &= 5b + 12b \\
 x &= 17b
 \end{aligned}$$

Example: Solve the following equation for y :
 $\frac{x}{5} + \frac{y}{3} = 5$ Multiply every term by the LCD, 15.

$$\begin{aligned}
 3x + 5y &= 30 \\
 3x - 3x + 5y &= -3x + 30 - 3x \\
 5y &= -33x \\
 \frac{5y}{5} &= \frac{-33x}{5} \\
 y &= \frac{-33x}{5}
 \end{aligned}$$

Example: Solve the following equation for h :

$$\begin{aligned}
 V &= \pi r h^2 \\
 \frac{V}{\pi r} &= \frac{\pi r h^2}{\pi r} \\
 \frac{V}{\pi r} &= h^2 \\
 h &= \sqrt{\frac{V}{\pi r}}
 \end{aligned}$$

Exercises: Solve the following equations for the indicated variable.

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|--------------------------------|----------------------------|-----------------------------------------|--------------------------------------|
| 1. $A = LW$ for L | 2. $I = prt$ for r | 10. $4x + 3a = 3x - 2a$ for x | 3. $P = 2L + 2W$ for W |
| 4. $x + y = 5$ for x | 5. $3x + y = 7$ for y | 11. $3(x + 2y) = 4$ for x | |
| 7. $A = \frac{a+b}{2}$ for a | 8. $A = \pi r^2$ for π | 13. $\frac{a}{3} = \frac{b}{3}$ for y | 14. $\frac{1}{2}(p - q) = m$ for m |

Answers:

- | | | |
|----------------------------|------------------------------|---------------------|
| 1. $L = \frac{A}{W}$ | 2. $r = \frac{I}{pt}$ | 3. $P = 2L + 2W$ |
| 6. $y = \frac{c - ax}{b}$ | 7. $\pi = \frac{A}{r^2}$ | 8. $y = 3 - 5 - 6x$ |
| 11. $x = \frac{4 - 6y}{3}$ | 12. $y = \frac{-5 - 6x}{18}$ | 13. $y = 3$ |

6. $ax + by = c$ for y

9. $V = \frac{1}{3}\pi r^2 h$ for h

12. $6(x + 3y) = -5$ for y

15. $\frac{3}{4}(2x + y) - \frac{1}{2}\pi r$
for y

14. $q = p - 2m$
15. $y = 7 - 3x$

$x = -5a$

4. $x = 5 - y$

5. $y = \frac{2 - 6x}{3}$

9. $3V = \frac{1}{2}h$

10. $h = \frac{2}{3}$