

Systems of Linear Equations

EXAMPLES

1. Solve

$\begin{array}{r} 4x + 2y = -6 \\ 5x + 5y = 10 \end{array}$	Multiply the first equation by -5 and multiply the second equation by 2 .
$\begin{array}{r} -20x - 10y = 30 \\ \underline{10x + 10y = 20} \\ -10x \qquad = 50 \end{array}$	Add the two equations.
$-10x = 50$	Solve for x .
$\begin{array}{r} x = -5 \\ 4(-5) + 2y = -6 \end{array}$	Let $x = -5$ in $4x + 2y = -6$.
$\begin{array}{r} -20 + 2y = -6 \\ 2y = 14 \end{array}$	Solve for y .
$y = 7$	The solution occurs when $x = -5, y = 7$.

The solution is $(-5, 7)$. The system is **consistent** and **independent**.

2. Solve

$\begin{array}{r} 2x + 3y = 6 \\ 4x + 6y = 12 \end{array}$	Multiply the first equation by -2
$\begin{array}{r} -4x - 6y = -12 \\ \underline{+4x + 6y = 12} \\ 0 = 0 \end{array}$	Add the two equations

The system has infinitely many solutions, as the second equation simply 2 times the first equation. Therefore, they are the same line, and will an infinite number of solutions. This is referred to as **consistent** and **dependent**.

3. Solve

$3x - 4y = 10$ $-6x + 8y = 20$	Multiply the first equation by 2
$6x - 8y = 20$ $\underline{-6x + 8y = 20}$ $0 = 40$	Add the equations

These equations have no solution. This is called **inconsistent** and means that the lines are parallel and will never intersect.

Summary

Consistent and independent	Perpendicular lines, one solution
Consistent and dependent	Same line, infinitely many solutions
Inconsistent	Parallel lines, no solution