

Systems of Linear Equations

EXAMPLE

1. Solve for the system of linear equations.

$$\begin{aligned}4x + 2y &= -6 \\5x + 5y &= 10\end{aligned}$$

SOLUTION

$$\begin{aligned}4x + 2y &= -6 \\5x + 5y &= 10\end{aligned}$$

Multiply the first equation by -5 and multiply the second equation by 2 .

Add the two equations.

$$\begin{array}{r} -20x - 10y = 30 \\ \underline{10x + 10y = 20} \\ -10x \qquad = 50 \\ -10x = 50 \end{array}$$

Solve for x .

$$x = -5$$

Let $x = -5$ in $4x + 2y = -6$.

$$4(-5) + 2y = -6$$

$$-20 + 2y = -6$$

Solve for y .

$$2y = 14$$

$$y = 7$$

The solution occurs when $x = -5, y = 7$.

The solution is $(-5, 7)$

The system is **consistent** and

We will eliminate y . **independent**

system.

EXAMPLE

2. Solve for the system of linear equations.

$$12x - 4y = 20$$

$$9x - 3y = 5$$

We will eliminate y .

Multiply the first equation by -3 and

SOLUTION

multiply the second equation by 4 .

Add the two equations.

$$12x - 4y = 20$$

$$9x - 3y = 5$$

$$-36x + 12y = -60$$

$$\underline{36x - 12y = 20}$$

Since use of the addition method has resulted in the case where all of the variables cancel and we are left with a false statement ($0 = -40$), we know that the system of equations has no solution.

$$0 = -40$$

This is an **inconsistent** system.

EXAMPLE

3. Solve for the system of linear equations

$$x - 5y = 3$$

$$-4x + 20y = -12$$

$$\underline{-4x + 20y = -12}$$

$$0 = 0$$

SOLUTION

We will eliminate x .

Multiply the first equation by 4 .

$$x - 5y = 3$$

$$-4x + 20y = -12$$

Add the two equations.

$$4x - 20y = 12$$

Since use of the addition method has resulted in the case where all of the terms cancel on both sides of the equals sign, leaving the true statement $0 = 0$, we know that there are infinitely many solutions.

$$\{(x, y) \mid y = 5x - 3\}$$

This is a **consistent** and **dependent** system.

Exercises: Solve for the following systems of linear equations

1. $3x + y = -1$ $2x - y = -5$

2. $5x - y = 5$
 $-10x + y = -8$

3. $2x - y = 10$
 $5x + y = 17$

4. $5x - y = 5$
 $-10x + y = 10$

5. $34x - y = 5$

$- + = -34xy \quad 3$

Answers:

1. $(-1,1)$ 2. \emptyset 3. $(1,3)$ 4. $\{(x,y) \mid 5 - 2y = 5\}$ 5. \emptyset