Translating Verbal Expressions Into Variable Expressions

Before you can begin to translate verbal expressions into variable expressions you must recognize the phrases which tell you to add, subtract, multiply or divide. Be especially careful with the phrases which indicate subtraction and division. These operations are not commutative and the order is very important.

"15 less than 12" means $12 - 15$ *not* $15 - 12$

In many problems there is more than one operation. It is important to be able to identify the main operation in a sentence. In general, the first word that you come to which tells you to add, subtract, multiply or divide will give you the main operation.

EXAMPLE: "the sum of 4 times x and y"

As the phrase "sum of" comes first, we know that the main operation is addition. If we are adding, we must have two and only two things to add. One is "4 times x," and the other is "y."

"the sum of 4 times x and y" translates to $4x + y$

Now we will change it slightly so that the main operation becomes multiplication.

EXAMPLE: "four times the sum of x and y"

Here the word "times" comes first so that the main operation is multiplication. We must have two and only two things to multiply. One is "4," and the other is "the sum of x and y"

"four times the sum of x and y" translates to $4(x + y)$

Note that it was necessary to use parentheses here to show the multiplication and to keep the sum of x and y together.

Let's put these two phrases and their translations next to each other so that we can compare them.

"the sum of four times x and y" = $4x + y$

"4 times the sum of x and y" = $4(x + y)$

See the difference?

To convince yourself that these are different, try this experiment. Substitute the value 2 for x and 3 for y and simplify.

$$4x + y = 4(2) + 3 = 8 + 3 = 11$$

$$4(x + y) = 4(2 + 3) = 4(5) = 20$$

In many problems it will be necessary to assign a variable to an unknown number.

EXAMPLE: “nine more than five times a number”

The word “number” refers to an unknown quantity, and a variable must be substituted. We will use the letter n.

“nine more than five times a number” translates to $5n + 9$

Note that if you had written $9 + 5n$ you would also be correct because addition is commutative. This would not work with “less than.”

$5n - 9$ is not the same as $9 - 5n$

Prove this to yourself by substituting the value 2 for n.

$$5n - 9 = 5(2) - 9 = 10 - 9 = 1$$

$$9 - 5n = 9 - 5(2) = 9 - 10 = -1$$

When the word “number” occurs more than once in a problem, you must use the <u>same variable</u> each time.

EXAMPLE: “ten less than a <u>number</u> divided by the sum of the <u>number</u> and one.”

We will use the variable x this time. The main operation is subtraction. “ten

less than” is “-10 ”

“a <u>number</u> divided by the sum of the <u>number</u> and one” $\frac{x}{x+1}$ is

Putting it all together, we have

$$\frac{x}{+1} - 10\,x$$

The last step in this section is to simplify the variable expression once you have made the translation. This means that you must perform any indicated operations and combine like terms.

EXERCISES: Translate and simplify if possible.

1. x less than 5

2. The product of -2 and n

3. Three times the sum of a number and four

4. The quotient of a number and 2 less than the number

5. The sum of a number squared and twice the number

6. x decreased by the product of 3 and x

7. Twice the sum of 2 and a number

8. One half a number minus two thirds of the number

9. The difference between the square of a number and 5 times the square of that number

10. Fifteen multiplied by two fifths of a number

KEY:

1. $5-x$

2. $-2n$

4. $\frac{n}{n-2}$

5. x^2+2x

7. $2(2+n) = 4+2n$

8. 1 2 1 — $x - x = -x$

10. $15\cdot\frac{2}{5}n = 6n$

2 3 6

3. 3(x+ = +4)3x 12 6. $x-3x=-2x$ 9. $x^2-5x^2=-4x^2$