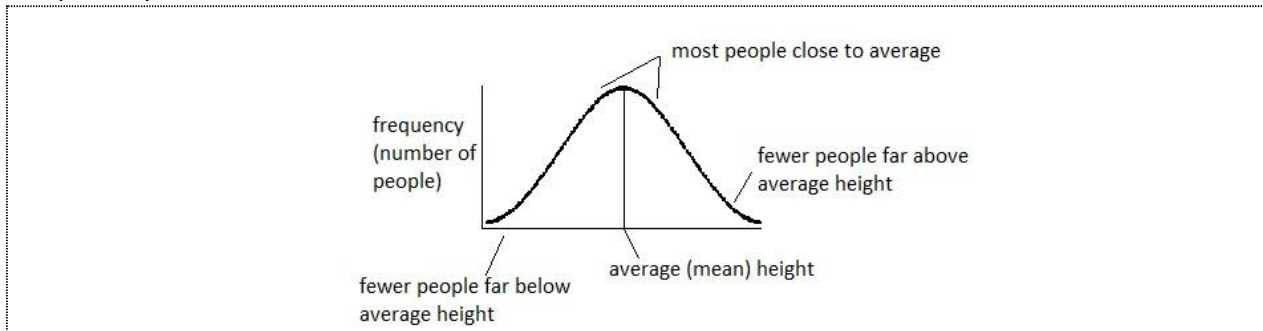


## Z-scores and the Empirical Rule

The heights of individuals within a certain population are normally distributed. That is, it will be found that most peoples' heights are clustered around an average height, and the farther a particular height is above or below the average, the less people we would find with that height. Graphically, it looks like this:



**A Z-score tells how many Standard Deviations a value is above or below the Mean.**

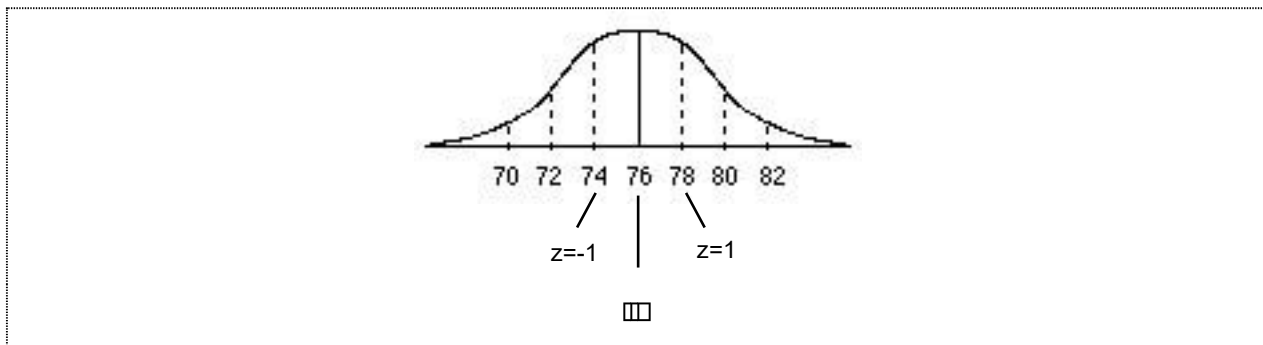
Suppose that the Mean height ( $\mu$ ) of college basketball players is 6 feet 4 inches (change the feet to inches; only one measuring unit is needed 76 inches =  $\mu$ ) with a Standard Deviation ( $\sigma$ ) of 2 inches.

- A height of 78" (X) is 2" above the Mean; 2" is one Standard Deviation; so the Z-score associated with 78" is positive 1.

$$\frac{X - \mu}{\sigma} = \frac{78 - 76}{2} = \frac{2}{2} = 1$$

- A height of 74" (X) is 2" below the Mean; 2" one Standard Deviation; so the Z-score associated with 72" is negative 1.

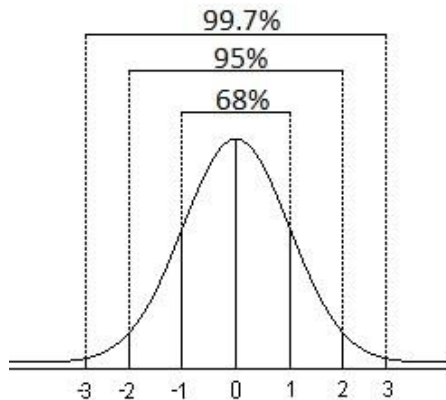
$$\frac{X - \mu}{\sigma} = \frac{74 - 76}{2} = -\frac{2}{2} = -1$$



**Empirical Rule or 68-95-99.7 Rule:**

The Empirical Rule says that:

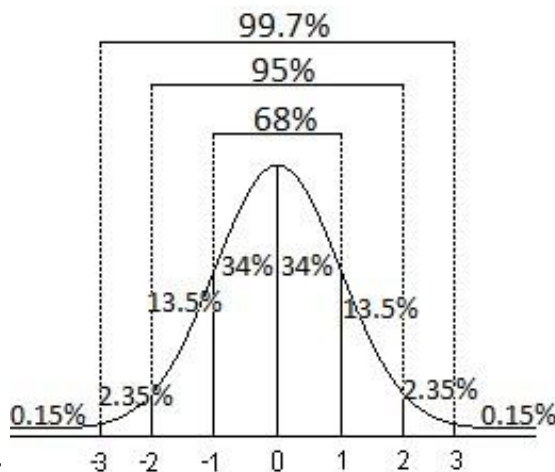
- Approximately 68% of the distribution will be within one standard deviation of the mean. This is  $Z = \pm 1$
- Approximately 95% of the distribution will be within two standard deviations of the mean. This is  $Z = \pm 2$
- Approximately 99.7% of the distribution will be within three standard deviations of the mean. This is  $Z = \pm 3$



It is easier to work with the Empirical Rule if the percentages are broken down evenly. The 68% can be split into 34% on each side of the Mean, so from the Mean to the First Z-score there will be 34% of the Distribution.

This can also be applied to the 95%. If it is split in half, there will be 47.5% between the Mean and the Second Z-score. To calculate the percentage between the First Z-score and the Second Z-score just subtract the 34% from the 47.5%:  $47.5\% - 34\% = 13.5\%$

This can be done to the Third Z-score and the 99.7%. The result of the breakdown will look



like this: