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LEARNING **COMMONS**

GRAPH THEORY

A **graph** consists of a finite set of points, called **vertices** (singular is *vertex*), and line segments or curves, called **edges**, that start and end at vertices. An edge that starts and ends at the same vertex is called a **loop.**

The **degree of a vertex** is the number of edges at that vertex. A loop connects a vertex to itself so it contributes 2 to the degree of the vertex. A vertex with an even number of edges attached to it is an **even vertex**. A vertex with an odd number of edges attached to it is an **odd vertex**.

Two vertices are **adjacent** if there is at least one edge connecting them.

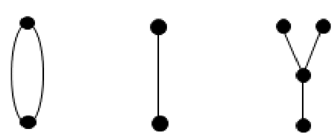
A **path** is a sequence of adjacent vertices and the edges connecting them. An edge can be part of a path only once.

Euler Paths and Circuits

An **Euler** (pronounced "oil·er") **path** is a path that travels **through every edge of a graph once and only once**. An Euler path that starts and ends on the same vertex is called an **Euler circuit**.

Euler's Theorem

How many Euler Paths and Circuits?
(It is all about the number of **odd** vertices.)



No odd vertices – at least one Euler path and circuit.

Exactly two odd vertices – at least one Euler path and NO Euler circuit.

More than two odd vertices – NO Euler path or Euler circuit.

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A graph is **connected** if for any two of its vertices there is **at least one path connecting them**. A connected graph consists of one piece. A graph that is not one piece is called **disconnected**.

A **bridge** is an edge that if removed from a connected graph would leave behind a disconnected graph.

A **tree** is a connected graph with no circuits. It has these properties:

- There is one and only one path joining any two vertices.
- Every edge is a bridge.
- A tree with n vertices has $n - 1$ edges.

A spanning tree is a subgraph that contains all of a connected graph's vertices, is connected, and contains no circuits.

Use Kruskal's Algorithm to find the **minimum spanning tree** of a weighted graph. Pick the smallest available edge but avoid creating circuits.

Hamilton Paths and Circuits

A **Hamilton path** is a path that passes **through each vertex of a graph exactly once**. A Hamilton path that starts and ends on the same vertex is called a **Hamilton circuit**.

A **complete graph** is a graph that has an edge between each pair of its vertices. A complete graph with n vertices has $(n - 1)$! Hamilton circuits.

Traveling salesperson problem

The **Brute Force Method** involves calculating the weights of <u>all possible Hamilton circuits</u>. It is used to find the Hamilton circuit in a complete weighted graph for which the sum of the weights of the edges is a minimum. That circuit is the optimal solution.

The **Nearest Neighbor Method** can be used to approximate the optimal solution by continually taking an edge with the smallest weight.