The Fundamental Counting Principle

Problems involving counting usually deal with ways of selecting objects, numbers, or people. Sometimes these selections involve a need to order and/or a need to repeat. Order in an experiment is essential when things, numbers, or persons must occupy a first position, a second position, etc. License plate characters and "combination" lock numbers imply a need for order or arrangement:

ABC 123 is a different license number from ABC 321.

Left 25-Right 13-Left 25 is a different lock "combination" from Left 25-Right 25-Left 13. (Note the repetition of 25)

Choosing 3 members for county commissioner from a field of 9 candidates involves no ordering and no repetition.

The Fundamental Counting Principle is used when **order is implied** or stated, and **repetition may or may not be allowed**.

The FCP uses the operation of multiplication. It involves drawing a "slot" for each possible outcome, filling in the number of possibilities for each outcome, and then multiplying across.

Example 1

Two fair dice are rolled. How many different ways can they land? A fair die can have 6 possible number outcomes: 1, 2, 3, 4, 5 or 6

Order underlies the experiment since a "2" on the 1st die and a "4" on the 2nd die is a different outcome from a "4" on the 1st and "2" on the 2nd. **Repetition is implied** in this experiment since the outcomes of the 2 dice are independent of each other.

1st die		2nd die		
6	•	6	=	36
(possible outcomes)		(possible outcomes)		Different outcomes possible

The table shows the 36 possible outcomes.

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	**2,4**	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	**4,2**	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Example 2

In how many ways can George, John, Charles, Mary, and Alice stand in a line so that boys and girls alternate? (Hint: a boy must occupy the first position since there are 3 boys, 2 girls).

Order matters because of the positioning of boy/girl

Repetition is not used since when a person is picked he/she cannot be chosen again.

Positions	1st		2nd		3rd		4th		5th		
	3	•	2	•	2	•	1	•	1	=	12
	total boys		total girls		boys left		girl left		boy left		possible lineups

The list shows the 12 Arrangements

George Mary John Alice Charles

George Alice John Mary Charles George Alice

Charles Mary John George Mary Charles Alice

John John Mary George Alice Charles

John Mary Charles Alice George

John Alice George Mary Charles

John Alice Charles Mary George

Charles Mary John Alice George

Charles Mary George Alice John

Charles Alice John Mary George

Charles Alice George Mary John

Example 3

How many different three course dinners can be ordered from a menu that has four choices for entree, six choices for vegetables, and three choices for desserts?

Order or arrangement **is implied** since we are choosing one of each from choices that are different from each other.

Repetition is not implied since a dinner will consist of three unique choices.

Entree		Vegetable		Dessert		
4	•	6	•	3	=	72
choices		choices		choices		different dinners