The T Confidence Interval of a Population Mean

T Intervals are similar to Z Intervals; however, instead of a Critical Z Value, a T Interval uses a Critical T Value, and uses the sample standard deviation, s, instead of the population standard deviation, σ.

$$T\ Confidence\ Interval = \bar{x} \pm t \times \frac{s}{\sqrt{n}}$$

Consider this example: The Interdepartmental Delivery Service coordinator wants to estimate the true mean number of interdepartmental letters employees receive per week. She takes a sample of 26 employees and obtains a mean of 6 letters per week with a standard deviation of 2.3 letters. She needs to be 99% confident of her estimate. Construct the Confidence Interval for the true mean for interdepartmental letters delivery per week.

Step 1: Find the Standard Error:

$S.\ E. = \frac{s}{\sqrt{n}}$ $\quad E. = \frac{2.3}{\sqrt{26}} \approx 0.451$ $\quad S.$

Step 2: Select Critical Value:

To find the Critical Value of a T distribution it will be easiest to use the Table at the end of a textbook (your teacher will provide a T table for tests, and exam). The Degrees of Freedom is calculated by subtracting one from the Sample Size; $DF = n - 1 = 26 - 1 = 25$. Match the Confidence Column with the Row that has the Degrees of Freedom.

Central Area		.80	.90	.95	.98	.99
Confidence Level		80%	90%	95%	98%	99%
Degrees of Freedom. n-1	21	1.32	1.72	2.08	2.52	2.83
	22	1.32	1.72	2.07	2.51	2.82
	23	1.32	1.71	2.07	2.5	2.81
	24	1.32	1.71	2.06	2.49	2.8

	25	1.32	1.71	2.06	2.49	2.79

Step 3: Compute the Margin of Error:

$$Margin\ of\ Error = t \times S.E.$$

$$Margin\ of\ Error = 2.79 \times 0.451$$

$$Margin\ of\ Error = 1.258$$

Step 4: Write out the Confidence Interval:

$$C.I. = \bar{x} \pm M.o.E.$$

$$C.I. = 6 \pm 1.258$$

$$(6 - 1.258, 6 + 1.258)$$

$$(4.742, 7.258)$$

Step 5: Interpret the Confidence Interval:

> "The Coordinator can be 99% confident that the average number of interdepartmental employee letters is between 4.742 and 7.258 letters per week."

Use the Texas Instruments calculator to find the confidence interval

Calculator Steps:

STAT > TESTS > 8: TInterval…

Inpt: Stats

$\bar{x}$: 6

Sx : 2.3
n : 26 C-Level:

.99 Calculate:

The Output:

(4.7427, 7.2573)

$\bar{x}$: 6

Sx = 2.3
n = 26