Matrix to Reduced Echelon Form, Step-by-Step

This handout accompanies the YouTube video - **https://www.youtube.com/watch?v=7piTfgtNa_U**

System of 3 equations

$2x + 5y + 4z = 12$ 2

$3x + y - 4z = 1$ 3

$x - 2y - 3z = 0$ 1

Create the matrix.

$$\left|\begin{array}{ccc|c} 5 & 4 & & 12\ 1 \\ 1 & -4 & & 1\ 0 \\ -2 & -3 & & 0\ 0 \end{array}\right|$$

The goal:

$$\left|\begin{array}{ccc|c} 0 & 0 & ? & \\ 1 & 0 & ? & \\ 0 & 1 & ? & \end{array}\right|$$

Organize your work like this to show step-by-step how you put a matrix in reduced row echelon form.

Updated matrix	Plan for this step	Calculator step
$\left\|\begin{array}{cccc} 2 & 5 & 4 & 12 \\ 3 & 1 & -4 & 1 \end{array}\right\|$ Verify output is correct	$R_1 \leftrightarrow R_3$	MATRIX >MATH >**C:rowSwap([A],1,3)**
1 -2 -3 0		STO→[A]
	$-3(R_1) + R_2 \rightarrow R_2$	
$\left\|\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 3 & 1 & -4 & 1 \\ 2 & 5 & 4 & 12 \end{array}\right\|$	-3 6 9 0 3 1 -4 1 0 7 5 1	MATRIX >MATH >**F:*row+(-3,[A],1,2)** Verify output is correct STO→[A]
	$-2(R_1) + R_3 \rightarrow R_3$	

1	-2	-3	0
0	7	5	1
2	5	4	12

-2	4	6	0
2	5	4	12

MATRIX >MATH >**E:*row+(-2,[A],1,3)**

Verify output is correct

0 9 10 12

STO→[A]

MATRIX > MATH > **F:*row(1/7,[A],2)**

1	-2	-3	0
0	7	5	1

$1/7(R_2) \rightarrow R_2$

Verify output is correct

0 9 10 12

STO→[A]

Change decimals MATH > ENTER > ENTER

1 -2 -3 0

0 1 .714 .142

to fractions

Verify output is correct

0 9 10 12

STO→[A]

Updated matrix

1	0	-11/7	2/7
0	1	5/7	1/7

0 9 10 12

1	-2	-3	
	0		
0	1	5/7	1/7

0 9

10 12

1	0	-11/7	2/7
0	1	5/7	1/7
0	0	25/7	75/7

$-9(R_2) + R_3 \rightarrow R_3$

0	-9	-45/7	-9/7
0	9	10	12
0	0	25/7	75/7

1	0	-11/7	2/7
0	1	5/7	1/7
0	0	1	3

$7/25(R_3) \rightarrow R_3$

1	0	0	5
0	1	5/7	1/7
		-2	-3

> MATH > **F:*row+(-9,[A],2,3)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

Plan for this step

Calculator step

$2(R_2) + R_1 \rightarrow R_1$

	2	10/7	2/7
0			0
1			
1	0	-11/7	2/7
0	0		

MATRIX > MATH > **F:*row+(2,[A],2,1)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

MATRIX > MATH > **E:*row(7/25,[A],3)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

1

$11/7(R_3) + R_1 \rightarrow R_1$

	0	0	
	1	0	-11/7	2/7
	1	0		
3			11/7	33/7
M				
A				
T			0	5
R				
I				
X				

MATRIX >MATH >**F:*row+(11/7,[A],3,1)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$-5/7(R_3) + R_2 \rightarrow R_2$

	0	0	-5/7	-15/7
	0	1	5/7	1/7
	0	1	0	-2

MATRIX >MATH > **F:*row+(-5/7,[A],3,2)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

The matrix is now in reduced-row-echelon form. Rewrite the system of equations and simplify.

1	0	0	5	
0	1	0	-2	$0x$
0	0	1	3	$0x$

$1x + 0y + 0z = 5 \quad \rightarrow \quad x = 5$

$+ 1y + 0z = -2 \quad \rightarrow \quad y = -2$

$+ 0y + 1z = 3 \quad \rightarrow \quad z = 3$

Write answer as an ordered triplet. $(5, -2, 3)$