Summary of the Four Cases of Partial Fraction Decomposition

Find the Partial Fraction Decomposition:

Case 1: a) All Linear Factors. b) No Repeating Factors **Ex:** $\frac{-x^2+30x-56}{x(x-7)(x+8)}$

1) Set up a separate fraction for each factor in the denominator and use a distinct variable for each numerator. The degree of the numerator factor should be one less than the degree of the denominator factor:

Set-up Solution:

$$\frac{A}{x} + \frac{B}{x-7} + \frac{C}{x+8} = \frac{-x^2+30x-56}{x(x-7)(x+8)}$$

2) Multiply every term in the Set-Up Solution by the LCD and solve the resulting equation for A, B and C:

Equation Set-Up: $-x^2 + 30x - 56 = A(x-7)(x+8) + Bx(x+8) + Cx(x-7)$

Simplify Equation Set-Up: $-x^2 + 30x - 56 = \boldsymbol{Ax^2 + Ax - 56A + Bx^2 + 8Bx + Cx^2 - 7Cx}$

Method 1: Note: a) Equate the coefficients to set up a System of Equations:

$$\begin{aligned} Ax^2 + Bx^2 + Cx^2 &= -1x^2 \\ Ax + 8Bx - 7Cx &= 30x \\ -56A &= -56 \end{aligned}$$

b) Set-up a matrix and solve using the calculator steps on the last page: $\begin{bmatrix} 1 & 1 & -3 \\ 1 & 8 & -7 \\ -56 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 30 \\ -56 \end{bmatrix}$

Method 2: Note: Find the zeros of factors of each denominator in the Set-up Solution to find A, B and C.

3) Substitute x = 0 , x = 7 and x = -8 to try to solve for A, B and C.

a) Substitute x = 0 and simplify to get: -56 = -56A Therefore: **A = 1**

b) Substitute x = 7 and simplify to get: 105 = 105B Therefore: **B = 1**

c) Substitute x = −8 and simplify to get: -360 = 120C Therefore: **C = -3**

Solution: $A = 1$, $B = 1$, $C = -3$ to get: $+\frac{\mathbf{1}}{\boldsymbol{x}}\ \frac{\mathbf{1}}{\boldsymbol{x-7}} + \frac{\mathbf{-3}}{\boldsymbol{x+8}}$

Find the Partial Fraction Decomposition:

Case 2: a) All Linear Factors. b) Some Repeating Factors **Ex:** $\frac{-7x^2+29x-18}{x(x-3)^2}$

1) Set up a separate fraction for each factor in the denominator including for all factors of the repeated factor and use a distinct variable for each numerator The degree of the numerator factor should be one less than the degree of the denominator factor:

Set-up Solution: $\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} = \frac{-7x^2+29x-18}{x(x-3)^2}$

2) Multiply every term in the Set-Up Solution by the LCD and solve the resulting equation for A, B and C:

Equation Set-Up: $-7x^2 + 29x - 18$ = A(x - 3)(x - 3) + Bx(x - 3) + Cx

Simplify Equation Set-Up: $-7x^2 + 29x - 18 = \boldsymbol{Ax^2 - 6Ax + 9A + Bx^2 - 3Bx + Cx}$

Method 1: a) Equate the coefficients to set up and solve a System of Equations:

$$\begin{aligned} Ax^2 + Bx^2 &= -7x^2 \\ -6Ax - 3Bx + Cx &= 29x \\ 9A &= -18 \end{aligned}$$

Method 2: Note: Find the zeros of factors of each denominator in the Set-up Solution to find A, B and C.

3) Substitute x = 0 and x = 3 to try to solve for A, B and C.

a) Substitute x = 0 to get: -18 = 9A Therefore, **A = -2**

b) Substitute x = 3 to get: 6 = 3C Therefore, **C = 2**

c) Substitute A = -2 and C = 2 to get:

$$-7x^2 + 29x - 18 = -2x^2 - 6x + 9 + Bx^2 - 3Bx + 2x$$

d) Equate the coefficients to find B: $-7x^2 = -2x^2 + Bx^2$

-5$x^2 = Bx^2$ Therefore: **B = -5**

Plug A, B and C into Set-Up Solution: $\frac{-2}{x} + \frac{-5}{x-3} + \frac{2}{(x-3)^2}$

Find the Partial Fraction Decomposition:

Case 3: a) No Repeating Factors. b) At least one Quadratic Factor. **Ex:** $\frac{3x^2 - x - 3}{(x-2)(x^2+3)}$

1) Set up a separate fraction for each factor in the denominator and use one distinct variab The degree of the numerator factor should be one less than the degree of the denominator factor le for the numerator of the linear factor and two distinct variables of the quadratic factor:

Set-up Solution: $\frac{A}{x-2} + \frac{Bx+C}{x^2+3} = \frac{3x^2-x-3}{(x-2)(x^2+3)}$

2) Multiply every term in the Set-Up Solution by the LCD and solve the resulting equation for A, B and C:

Equation Set-Up: $3x^2 - x - 3$ = A($x^2 + 3$) + (Bx + C)(x - 2)

Simplify Equation Set-Up: $3x^2 - x - 3 = \mathrm{A}x^2 + 3A + Bx^2 - 2Bx + Cx - 2C$

Method 1: a) Equate the coefficients to set up and solve a System of Equations:

$$\begin{array}{lllll} Ax^2 + Bx^2 & & & = & 3x^2 \\ & -2Bx & + Cx & = & -1x \\ 3A & & -2C & = & -18 \end{array}$$

Method 2: Note: Find the zeros of factors of each denominator in the Set-up Solution to find A, B and C.

3) Substitute x = 2 to try to solve for A, B and C

a) Substitute x = 2 and simplify to get: 7 = 7A Therefore, **A = 1**

b) Substitute A = 1 to get: $3x^2 - x - 3 = x^2 + 3 + Bx^2 - 2Bx + Cx - 2C$

c) Equate the coefficients to find B: $3x^2 = x^2 + Bx^2$ Therefore, **B = 2**

d) Substitute A = 1 , B = 2 and equate the coefficients to find C: $-x = -4x + Cx$

3x = Cx Therefore: **C = 3**

Solution: $A = 1$, $B = 2$, $C = 3$ to get: $\frac{1}{x-2} + \frac{2x+3}{x^2+3}$

Case 4: a) Some Repeated Quadratic Factors.

Set up the fractions for the Partial Fraction Decomposition Ex: $\frac{2x^4 + x^3 + 8x^2 - 3x + 8}{x(x^2+2)^2}$

1) Set up a separate fraction for each factor in the denominator and use one distinct variable for the numerator of the linear factor and two distinct variables for each factor of the quadratic factors. The degree of the numerator factor should be one less than the degree of the denominator factor. Do not solve:

Set-up Solution: $\frac{2x^4 + x^3 + 8x^2 - 3x + 8}{x(x^2+2)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2}$

Practice Problems:

For each rational expression, set up and solve (except #4) a Partial Fraction Decomposition:

1) $\frac{x+32}{x(x-8)}$ 2) $\frac{2x^2+x+2}{x^2(x+1)}$ 3) $\frac{2x^2-7x-12}{(x+2)(x^2+1)}$ 4) $\frac{-3x^4-14x^2+8x-27}{x(x^2+3)^2}$

Answer Key:

1) **Set-up equation**: $\frac{A}{x} + \frac{B}{x-8}$ **Solution**: A = -4 , B = 5

2) **Set-up equation**: $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ **Solution**: A = -1 , B = 2, C = 3

3) **Set-up equation**: $\frac{A}{x+2} + \frac{Bx+C}{x^2+1}$ **Solution**: A = 2 , B = 0 , C = -7

4) **Set-up equation**: $\frac{A}{x} + \frac{Bx+C}{x^2+3} + \frac{Dx+E}{(x^2+3)^2}$

Graphing Calculator Steps for Solving a System of Equations

1) 2nd – Matrix
2) Edit
3) Enter Matrix Dimensions (including the solution column)
4) Enter Matrix Entries
5) 2nd Quit
6) 2nd Matrix
7) Math
8) B: rref

9) 2^{nd} Matrix (Select the Matrix that the data was entered into)

10) Enter