

Polar and Rectangular Coordinate Conversions

Polar Coordinate System – Any ordered pair written in the form of (r, θ) where r is the r radius from the Origin point O to a fixed point P and θ is the angle between the Polar Axis and the segment \overline{OP} .

Rectangular Coordinate System – Any ordered pair that can be written in the form of (x, y) where x is the horizontal component and y is the vertical component of the point.

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

Converting from Polar to Rectangular Coordinates:

Example: Find the Rectangular Coordinates for the point that has Polar Coordinates $(2, 60^\circ)$.

Solution: $x = r \cos \theta$ and $y = r \sin \theta$

$$\begin{aligned} x &= 2 \cos 60^\circ & y &= 2 \sin 60^\circ \\ &= 2 \times \frac{1}{2} & &= 2 \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$= 1 \qquad \qquad = \sqrt{3}$$

The Rectangular Coordinates for the point that has Polar Coordinates $(2, 60^\circ)$ is $(1, \sqrt{3})$

Converting from Polar Coordinates to Rectangular Coordinates:

Given $r^2 = x^2 + y^2$ and $\tan \theta = \frac{y}{x}$

Example: Find the Polar Coordinates for the point that has Rectangular Coordinates $(3, 3)$. **Solution:**

$$r^2 = x^2 + y^2$$

Given: $r^2 = 3^2 + 3^2$

$$\tan \theta = \frac{y}{x}$$

$$r^2 = 9 + 9$$

$$\tan \theta = \frac{3}{3}$$

$$r^2 = 18$$

$$\tan \theta = 1$$

$$r = \sqrt{18} = 3\sqrt{2}$$

$$\tan^{-1}(1) = 45^\circ$$

The Polar Coordinates for the point that has Rectangular Coordinates $(3, 3)$ is $(3\sqrt{2}, 45^\circ)$.

Example: Express the following equations in Polar coordinates (Solve for r): $y^2 = 2x$

Solution:

Step 1: $y^2 = (r\sin\theta)^2$ **and** $2x = 2r\cos\theta$

Step 2: $r^2(\sin\theta)^2 = 2r\cos\theta$

Step 3: Solve for r: $r = \frac{2\cos\theta}{(\sin\theta)^2}$

$$r = 2 \frac{\cos\theta}{\sin\theta} \frac{1}{\sin\theta} \quad \mathbf{r = 2\cot\theta\csc\theta}$$

Example: Express the following Polar equations in Rectangular Coordinates: $r = 5 \csc \theta$

Solution:

Step 1: $r = \frac{5}{\sin\theta}$

Step 2: $r\sin\theta = 5$

Step 3: $y = r\sin\theta = 5$ **$y = 5$**

Practice Exercises:

Find the rectangular coordinates for the point that has the given polar coordinates (Round to two decimal places):

1) $(4, 80^\circ)$

2) $(-2, 150^\circ)$

3) $(7, 33^\circ)$

Find the polar coordinates for the point that has the given rectangular coordinates (Round to two decimal places):

4) $(-3, 4)$

5) $(10, -2)$

6) $(5, 7)$

Express the following equation in Polar coordinates:

7) $2x^2 = y$

Express the Polar Equation in Rectangular Coordinates:

8) $r = 4\csc\theta$

Solutions:

1) $(0.69, 3.94)$

2) $(1.73, -1)$

3) $(5.87, 3.81)$

4) $(5, 126.87)$

5) $(10.20, 149.97)$

6) $(8.60, 54.46)$

7) $r = \frac{1}{2} \tan\theta\sec\theta$

8) $y = 4$