Polynomial Division Methods- Synthetic and Long Division

Method 2: Synthetic Division Example: $F(x) = 6x^4 - 7x^3 - 37x^2 + 8x + 12$ (factor)

Step 1: All choices for a root come from factors of $\underline{+}12/6$: $\underline{+}\frac{1,2,3,4,6,12}{1,2,3,6}$; the integer choices: $\underline{+}1,2,3,4,6,12$. To save some time, find some possible zeros using the calculator. Hint: to find a quick zero, put the function into $\boxed{y=}$ on the calculator, check the table to see if there is an integer zero (y-value = 0 in the table) In this case we see x = -2 and 3 have a value of 0 for the y.

Step 2: Set up your "table" which should look like an upside down division sign. Put the coefficients of the function inside, inserting a zero if any position is vacant. Put your test value on the outside.

-2	6	-7	-37	8	12

Step 3: bring down the first coefficient on the inside

-2	6	-7	-37	8	12
	6				

Step 4: multiply the outside number by the number you just brought down and insert this inside the division sign.

	6	-7	-37	8	12
		-12			
-2					
	6				

Step 5: add the numbers in the second column.

-2	6	-7	-37	8	12
		-12			
	6	-19			

Step 6: repeat step 5 & 6 until you reach the end.

-	6	-7	-37	8	12
		-12	38	-2	-12
	6	-19	1	6	
2					
					0

When you get a zero in the last position it is great, it means you have no remainder so -2 is a zero! x = -2 gives us a factor of x + 2.

Step 7: your function is now $(x + 2)(6x^3 – 19x^2 + x + 6)$. You can continue to use synthetic division to keep breaking down the polynomial or factoring. You can use the 3 that we found in step one as a possible zero. You can use the numbers at the bottom as your new coefficients to insert inside.

3

6	-19	1	6	18	-3		
			-6	6	-1	-2	0 (another root since no remainder)

Once you complete this step, you now have f(x) = (x + 2)(x - 3)(**$6x^2$ - x - 2).** Since we now have a quadratic (2^{nd} degree), we can factor or use the quadratic formula to find the other potential zeros. **$6x^2$ - x - 2** = (2x + 1)(3x - 2)

Thus our original polynomial: F(x) = $6x^4 - 7x^3 - 37x^2 + 8x + 12$ = **(x + 2)(x - 3)(2x + 1)(3x - 2).**

Method 1: Long Division Example: F(x) = $6x^4 - 7x^3 - 37x^2 + 8x + 12$ (factor)

Since we know that -2 and 3 are possible zeros from the first method, let's start with one of those.

$6x^4 - 7x^3 - 37x^2 + 8x + 12$ x + 2 Step 1: we need to find out what we have to multiply x by to make it match $6x^4$. x( ) = $6x^4$. You can divide to see that the ( ) needs to be $6x^3$. This goes on top of the division sign.

$6x^3$

$6x^4 - 7x^3 - 37x^2 + 8x + 12$

x + 2 Step 2: distribute the $6x^3$ times all of the terms in the front and put this result inside of the division sign. $6x^3(x + 2) = 6x^4 + 12x^3$. Be sure to line the terms up with the ones with matching exponents on the inside.

$6x^4 + 12x^3$

$6x^3$

x + 2 $6x^4 - 7x^3 - 37x^2 + 8x + 12$ Step 3: Draw a line under the terms you just added. We need to subtract at this point so change all of their signs. $-(6x^4 + 12x^3)$

$6x^3$

x + 2 $6x^4 - 7x^3 - 37x^2 + 8x + 12$ Step 4: add the columns and put the result below the line.

$\underline{-6x^4 - 12x^3}$

$0 - 19x^3$

$6x^3 - 19x^2$

$6x^4 - 7x^3 - 37x^2 + 8x + 12$ x + 2 Step 5. Bring down the next term in the polynomial you wrote at the top.

$\underline{-6x^4 - 12x^3}$

$- 19x^3 - 37x^2$

$- \underline{19x^3 - 38x^2}$

$6x^3 - 19x^2$

$6x^4 - 7x^3 - 37x^2 + 8x + 12$ Step 6: repeat step 1 – 5 until you don't get any more x's

x + 2 when you subtract. Or until you get a remainder with a lower degree than your divisor.

$\underline{-6x^4 - 12x^3}$

$- 19x^3 - 37x^2$ $\underline{-(-19x^3 - 38x^2)}$ then, draw the line and change the sign, add the columns, x^2 bring down the next term

$6x^3 - 19x^2 + x$

x + 2 $6x^4 - 7x^3 - 37x^2 + 8x + 12$

$\underline{-6x^4 - 12x^3}$

$- 19x^3 - 37x^2$

$\underline{\quad 19x^3} \; \underline{+ 38x^2}$ then, draw the line and change the sign, add the columns, bring down the next term

$x^2 + 8x$

$\underline{\cancel{-x^2 + 2x)}}$

$6x + 12$

$\underline{-6x + 12)}$ then, draw the line and change the sign, add the columns

0 the zero says that there is no remainder.

Thus we now have: $(x + 2)(6x^3 - 19x^2 + x + 6)$. If you continue breaking down the larger polynomial, you will eventually get all the factors. You can use long division, synthetic division or factoring to find the rest of the zeros. Check method 2 on the other side for a complete set of answers. Both methods should give the same answers!