Vector Algebra

Vector notation is designed to not just give you a point in space but a directional vector. They will usually look like <2,5>. This is giving you a vector that starts at the origin and continues in the direction of the (x, y) point of (2,5). Vectors can also be written in the form: 2***i*** + 5***j*.** The i component is always x and the j is y.

<u>Basic Vector Operations and Rules</u> (let ***u*** = <a,b> and ***v*** = <c,d>)

Magnitude of a vector: $|\mathbf{v}| = \sqrt{a^2 + b^2}$; length of the vector

Addition Rules: ***u*** + ***v*** = <a,b> + <c,d> = <a +c, b + d>

Scalar Multiplication: c***u*** = c<a,b>= <ca, cb>

Examples of addition and subtraction with vectors, scalar multiplication: ***Let:***

U = <2,3>, ***W*** = <-4, 6>

1) ***U*** + ***W*** = < 2 – 4, 3 + 6> = **< -2, 9> or -2i + 9j**
2) ***W*** – ***U*** = <-4, 6> - <2,3> = < -4 – 2, 6 – 3> = **< -6, 3> or -6i + 3j**
3) 2***U*** – 3***W*** = < 4, 6> - <-12, 18> = < 4 – (-12), 6 – 18> = **< 16, -12> or 16i – 12j**

You Try (1): find 4***U*** – 2***W***

<u>Unit Vector- magnitude is 1</u>

Example of converting a vector to a unit vector: **v** = <2,5>

1) Find the magnitude ❼ $\sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$
2) $\boldsymbol{U} = \frac{1}{|v|}\boldsymbol{v} = \boldsymbol{<}\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\boldsymbol{>}$
3) This would give a unit vector (length of 1) in the direction of **v.**

You Try(2): find the unit vector for the vector ***v*** = **<**-1, 5**>**

<u>Dot Product</u>

The dot product is used to find the angle between two vectors. The result will always be a scalar (a number not a vector).

Definition: if ***u*** = <a,b> and ***v*** = <c,d> then ***u* · *v*** = ac + bd

Example: ***u*** = <2,3>, ***v*** = <-4, 6>, then ***u*·*v*** = 2 *x* -4 + 3 *x* 6 = -8 + 18 = 10 You

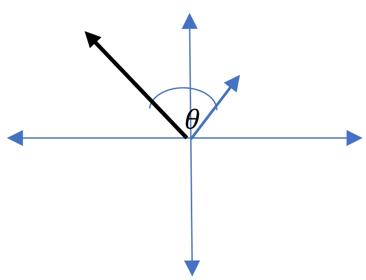
Try(3): u = <1, 5>, v = <-2, 1> solve for ***u*·*v***

Find the angle between two vectors (application of the dot product)

$$\cos\theta = \frac{u \,.\, v}{|u||v|}$$

So if ***U*** = <2,3>, ***V*** = <-4, 6>, then:

$\cos\theta = \frac{<2,3> \cdot <-4,6>}{|\sqrt{13}||\sqrt{52}|} = \frac{-8+18}{26} = \frac{10}{26} = \frac{5}{13}$; thus $\cos\theta = \frac{5}{13}$; so $\theta = 67.38°$.

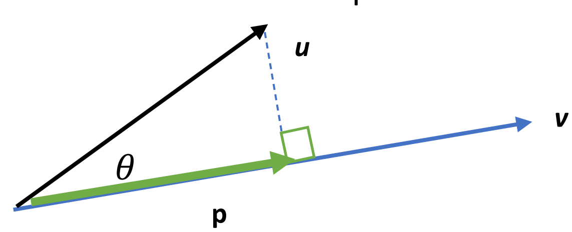


On another note, two vectors are said to “orthogonal” (at a right angle or 90° apart) if $\boldsymbol{u} \cdot \boldsymbol{v} = 0$

You Try(4): find the angle between ***U*** = <-2,1> and ***V*** = <-4, 3>

Finding the scalar component of *u* on *v*

An important use of the dot product is to determine the projection of one vector onto another if they share a common initial point.



If θ is the angle between vectors u and v, then the scalar component of u on v is given by:

$$\text{comp}_v\, \text{u} = \frac{u \,.\, v}{|v|}$$

So if ***u*** = <2,3>, ***v*** = <-4, 6>, then

$\text{comp}_v\, \text{u} = \frac{<2,3> \cdot <-4,6>}{|\sqrt{13}|} = \frac{10}{\sqrt{13}} =$ 2.77 to 2 significant figures.

One application of this is Work which can be defined as:

$$\text{comp}_d\, F\, |d| = \frac{F \cdot d}{|d|}\, |d| = F \cdot d$$

Answers to You Try's: <16, 0> 2) ***U*** = **<** $\frac{-1}{\sqrt{26}}$, $\frac{5}{\sqrt{26}}$ **>** 3) 3 4) 26.57 1) °