

## Verifying trigonometric identities

Process: make one side look exactly like the other using a combination of trigonometric identities and algebra. You can work with only one side at a time.

### 1. Algebra techniques utilized

a. "FOIL"ing      example 1       $(\cot x - \csc x)(\cos x + 1) = -\sin x$

b. "FOIL"ing      example 2       $\frac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$

c. distribution       $\sec t \csc t (\tan t + \cot t) = \sec^2 t + \csc^2 t$

d. Common denominator       $2 \sec x = \frac{1}{\sec x + \tan x} + \frac{1}{\sec x - \tan x}$

### 2. Conjugate

$$\frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

### 3. Substitution of identity

$$\sin^2 x + \cos^2 x + \tan^2 x = \sec^2 x$$

### 4. Turn all functions into sin x and cos x

$$\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$$

If all else fails, turn everything into sine x and cosine x and see what happens! Usually there is lots of algebra between using the trig functions. You have to be very familiar with the basic functions.

### Basic Functions

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

The last two can be obtained by dividing the first either by sine squared x or cosine squared x. Might also look like cosine x = 1 minus sine squared x or 1 = secant squared x - tangent squared x

## Examples

Worked out (remember, work with only one side until it looks like the other)

1.  $(\cot x - \csc x)(\cos x + 1) = -\sin x$  *(working with left side since more complicated)*


$= \cot x \cos x + \cot x - \csc x \cos x - \csc x$  FOIL the binomials

$= \frac{\cos x}{\sin x} \cos x + \frac{\cos x}{\sin x} - \frac{1}{\sin x} \cos x - \frac{1}{\sin x}$  insert  $\sin x / \cos x$  identities

$= \frac{\cos^2 x}{\sin x} + \frac{\cancel{\cos x}}{\cancel{\sin x}} - \frac{\cancel{\cos x}}{\cancel{\sin x}} - \frac{1}{\sin x}$  simplify

$= \frac{\cos^2 x - 1}{\sin x}$  cancel like terms

$= \frac{(1 - \sin^2 x) - 1}{\sin x}$  identity; eliminate  $\cos x$  term since not in answer

$= \frac{-\sin^2 x}{\sin x} = -\sin x$   reduce


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2.  $\frac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$  *(working with left side since more complicated)*

$= \frac{\sin^2 t + 2 \sin t \cos t + \cos^2 t}{\sin t \cos t}$  FOIL out the top

$= \frac{1 + 2 \sin t \cos t}{\sin t \cos t}$  combine  $\sin^2 x + \cos^2 x = 1$

$= \frac{1}{\sin t \cos t} + \frac{2 \sin t \cos t}{\sin t \cos t}$  separate fraction since final answer doesn't have one

$= \csc t \sec t + 2$   use reciprocals and reduce fraction